8. In the present paper, Professor Gronwall proves the following theorems:

I. When the analytic function $z = f(x) = a_0 + a_1 x + \cdots + a_n x^n + \cdots$ effects the conformal representation of the circle $|x| < 1$ on a simple (that is, simply connected and nowhere overlapping) region in the z-plane, the area of this region not exceeding A, then, for $|x| \leqq r < 1$,

$$|f(x)| \leqq \sqrt{\frac{A}{\pi} \cdot \log \frac{1}{1 - r^2}} \quad \text{and} \quad |f'(x)| \leqq \sqrt{\frac{A}{\pi}} \cdot \frac{1}{1 - r^2},$$

and these upper boundaries of $|f(x)|$ and $|f'(x)|$ cannot be replaced by any smaller ones. Less accurate limitations have been given by Koebe and Courant.

II. When $z = f(x) = 1/x + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots$ effects the conformal representation of the circle $|x| < 1$ on a simple region in the z-plane containing the point at infinity, then $|f(x)| < 9/4r$ for $|x| = r < 1$. A less accurate limitation has been given by Fricke.

F. N. Cole,
Secretary.

THE TWENTY-SIXTH REGULAR MEETING OF THE SAN FRANCISCO SECTION.

The twenty-sixth regular meeting of the San Francisco Section of the Society was held at the University of California on October 24, 1914. Twenty-two persons were present, including the following members of the Society:

Professor R. E. Allardice, Dr. B. A. Bernstein, Professor H. F. Blichfeldt, Professor C. E. Brooks, Dr. Thomas Buck, Professor L. E. Dickson, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. Frank Irwin, Professor D. N. Lehmer, Professor J. H. McDonald, Professor W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, Professor E. W. Ponzer.

The chairman of the Section, Professor Manning, presided at the opening of the meeting; the chairman-elect, Professor Haskell, then took the chair. The following officers were elected for the ensuing year: chairman, Professor Haskell; secretary, Dr. Buck; programme committee, Professors Manning and Blichfeldt, and Dr. Buck.

It was voted to omit the spring meeting of the Section because of the summer meeting of the Society to be held in San Francisco, and to hold the fall meeting at Stanford University at a date to be determined later.

The question of a colloquium in connection with the summer meeting in 1915 was discussed, and it was decided to petition the Council to arrange for such a colloquium.

The members took luncheon together as usual.

The following papers were presented at this meeting:

(1) Dr. FRANK IRWIN: "Relation between the zeros of a rational integral function and its derivative."

(2) Professor M. W. HASKELL: "The maximum number of stationary points of algebraic curves."

(3) Professor D. N. LEHMER: "Some further results in the theory of the continued fraction representing the surd $R^{1/\lambda}$" (second preliminary report).

(4) Professor L. E. DICKSON: "On formal and modular invariants."

(5) Professor L. E. DICKSON: "On modular curves."

Abstracts of the papers follow below.

1. The proposition that the roots of the derivative of a polynomial lie, in the complex plane, inside the smallest convex polygon containing the roots of the polynomial itself is well known; but Hayashi's proof recently published in the *Annals of Mathematics* is the first, so far as known, not based on dynamical conceptions. Dr. Irwin points out that an extremely simple proof of this kind may be given.

2. Professor Haskell shows that the maximum number of cusps possible for a plane curve of order m is the greatest integer in $m(m - 2)/3$; and that in every case there is a self-dual curve with this maximum number of cusps. The cases $m = 4$ and $m = 6$ are exceptions, the maximum number of cusps being 3 and 9 respectively.

In the case of space curves, the Plücker equations can be satisfied by the same maximum number of stationary points, but it can be shown that the curves are then either plane curves or reducible.

3. Representing the complete quotient in the expansion of $R^{1/\lambda}$ in a continued fraction by

$$\frac{P_n + A_n R^{1/\lambda} + B_n R^{2/\lambda} + C_n R^{3/\lambda} + \cdots + K_n R^{(\lambda-2)/\lambda} + L_n R^{(\lambda-1)/\lambda}}{Q_n}$$

and the corresponding convergent by α_n/β_n, Professor Lehmer has obtained certain interesting inequalities connecting the P's and Q's, which show that there can be only a finite number of P's which have the same value. In a former paper it was shown that the Q's satisfy the indeterminate equation

$$(-1)^{n-1} Q_n = \alpha_n^\lambda - R\gamma_n^\lambda,$$

and by a general theorem due to Axel Thue (Christiania, *Videnskabs-Selskabet Skrifter*, 1908, No. 3), there can be only a finite number of Q's having the same value in the expansion. This important result has not yet been derived from the discussion of the continued fraction itself.

4. The first paper by Professor Dickson gave a survey of the main results in the theory of invariants arising in the theory of numbers. Special attention was given to the construction of formal modular invariants from the geometrical standpoint developed in the October number of the *Transactions*.

5. The second paper by Professor Dickson related to the theory of modular cubic and quartic curves for the interesting case in which the modulus is 2. Such a quartic curve has at most seven bitangents (and aside from special cases exactly seven) whose intersections are either singular points or points with indeterminate polars. In general, all such points are intersections of bitangents. The equivalence of two quartic curves can be decided from a knowledge of their real points, their singular points, and their points with indeterminate polars.

THOMAS BUCK,
Secretary of the Section.

MODULAR INVARIANT PROCESSES.

BY PROFESSOR O. E. GLENN.

(Read before the American Mathematical Society, September 8, 1914.)

Introduction.

LET $f = a_0 x_1^n + \cdots$ be an ordinary algebraical quantic in m variables. Suppose that it is subjected to linear trans-