Problèmes d'Analyse mathématique. By E. FABRY. Paris, Libraire Scientifique A. Hermann et Fils, 1913. 460 pp.

THE primary object of this collection of problems seems to be to furnish material in which French student candidates preparing for the examinations for certificates in differential and integral calculus might be interested. A list of problems (279) given at examinations within the last twelve years and at fourteen different examination centers scattered throughout France occupies the first 66 pages of the book. The detailed solutions of the problems proposed fill quite completely the remainder of the volume. The reviewer has checked a few—not many—of these solutions.

The 279 problems in the twelve chapters are distributed in numbers from 5 to 35 over the field of analysis as follows: Two chapters on quadratures and their geometric applications. One on line integrals (complex variable, analytic functions). Four on differential equations and their applications to plane curves, space curves, and surfaces, including the determination of geodesics, asymptotic lines, and lines of curvature. A chapter on ruled surfaces (developable). Two chapters on partial differential equations and their geometric applications. One each on total differentials and elliptic functions.

Some of the problems—a few—might be included in the lists of "harder problems" in calculus texts designed for the American college sophomore; more could be solved with much labor in algebraic and trigonometric reductions by a class in the advanced calculus, but many would undoubtedly have to be "left for later consideration." Certainly, if the candidate passed an examination on a similar list he might well feel that he deserved his certificate.

ERNEST W. PONZER.

Cours de Géométrie infinitésimale. By G. DEMARTRES. Paris, Libraire Gauthier-Villars, 1913. x + 455 pp.

EVEN a cursory inspection of this volume would lead one to conclude that it was never intended to rival the classic works of Bianchi or Darboux. One easily comes to the conclusion, especially after reading the excellent preface by Appell, that the book is intended to be a text, not a reference book to accompany lectures, on a subject which, according to the reviewer's opinion, is sadly neglected in mathematical courses offered in this country. A course on differential geometry after the plan of the book under review or that of Scheffer's "Anwendung der Differential und Integral-rechnung auf Geometrie," with which one is continually tempted to make comparisons, would help round out any undergraduate course in mathematics offered. Incidentally it would serve as a safe stepping-stone from the calculus to the maze of analysis into which the student too often is plunged headlong.

The correct pedagogical principle, that by far the majority of students feel constrained to make use of geometric interpretations of analytic processes in order to understand the latter thoroughly, is kept constantly in the foreground. In fact it is with this purpose in mind that the whole of the first part of the book (134 pages) is developed along entirely geometric lines. Here the geometry of the infinitesimal by no means plays the minor rôle it too often is assigned, if it is not neglected entirely—yes, even in treatises on geometry! The theory of infinitesimals and its application to the problems of curvature, order of contact, special curves, kinematics, etc., is developed geometrically. So also for space curves and surfaces. Definite notions are given concerning the nature of curvature, principal normal, tangent plane, osculating sphere, asymptotic and geodesic lines, etc., and theorems are proved entirely from the geometric side.

If it should be that the analysis because of its generality, rigor, and art of being conclusive appeals to the student with greater force, then he might well begin the book with the second part; for the last three of the four parts into which it is divided emphasize the analysis. Or he could compare the analytic treatment of the subject with the geometric given in Part I—a comparison which would certainly help fix its principles. However, as far as possible, an endeavor is made to hold fast to geometric interpretation no matter how analytical the treatment becomes. As an aid in this direction may be mentioned the general plan which is carried throughout the last three parts. This consists in expressing the coordinates of a moving point by a power series in terms of the displacement along a curve. A proper choice of the axes at the origin of the moving point allows a geometric interpretation of much of the analysis at each instant in terms of simple geometric relations.

The theory of lines, straight or curved, in a plane, free to move, or situated on a surface, together with the usual and some special theorems accompanying it, is developed in Parts II and III. In Part IV curvilinear coordinates are used in the problems and theorems of space curves and surfaces. Here is also included the application of line coordinates to the general theory of surfaces of the second order.

A list of 136 exercises, to some of which is added the suggestion that they be solved geometrically, is given at the close of the book.

It is to be regretted that the volume does not possess the high typographical tone and finish which characterize Scheffer's text.

ERNEST W. PONZER.

Untersuchungen über die im Schlusswort des Lie'schen Werkes "Geometrie der Berührungstransformationen" angedeuteten Probleme. Von PHILIPP ENGELHARDT (Inaugural-Dissertation, Würzburg). Leipzig, B. G. Teubner, 1910. 65 pp.

THE closing lines of Lie's Geometrie der Berührungstransformationen forecasted that a second volume, which never appeared, would treat the theory of differential equations from the standpoint of contact transformations and continuous groups. One may well hope that this field will soon receive its fair share of attention and be extensively developed beyond the elements established by Lie.

The thesis under review might be classed with fundamental literature having this tendency. The author's methods are, for the most part, those presented by Lie in chapter 14. The integral surfaces of a non-linear partial differential equation of the first order, $F(x, y, z, p, q) = 0$, bear an aggregate of ∞^3 exceptional curves called characteristics (Geometrie der Berührungstransformationen, pages 261, 498). Lie's summary of his last chapter is in the form of statements of six problems which are there solved. These are determinations of differential equations $F = 0$ defined by definite conditions imposed upon the characteristics or upon the normals to integral surfaces. Problem 2 is the determination of the equations $F = 0$ whose characteristics are lines of curvature upon all integral surfaces. Problem 4 is the determination of the equations $F = 0$ whose integral surfaces have for normals precisely lines of a given line complex. This thesis is devoted to twelve problems obtained by combining Lie's six problems. Thus problem (2, 4) of the thesis is the deter-