may elicit some dissertations that may bring light on these vital problems.

## SHORTER NOTICES.

*Die Mathematik im Altertum und Mittelalter.* By H. G. ZEUTHEN. Erster Abschnitt: *Entstehung und Entwicklung der Zahlen und des Rechnens.* B. G. Teubner, Leipzig und Berlin, 1912. 95 pages. Price unbound, 3 Marks.

THIS little work is a portion of Teil III of the volume entitled Die mathematischen Wissenschaften, in the work edited by Professor Hinnenberg, Die Kultur der Gegenwart. Teil III has for its title Die mathematischen und naturwissenschaftlichen Kulturgebiete, and the part relating to mathematics is under the editorial supervision of Professor Klein.

No opportunity was afforded, in the brief space allotted to Professor Zeuthen, for any new contribution to the history of mathematics. Therefore all that can be expected is a mere résumé of the leading contributions to the science in ancient and medieval times. Professor Zeuthen first treats of early arithmetic, beginning with the primitive number systems, passing to the early mechanical methods of computation, setting forth the difficulties of notation in ancient times, tracing rapidly the development of our numerals, and making clear the obstacles met by all early peoples in the handling of fractions. He then considers the applications of number to commerce, astronomy, mysticism, and puzzle problems, showing the relation of this work to the primitive algebra of the Egyptians.

He next takes up the geometry of the Egyptians and Greeks. While giving only a cursory glance at the development of the science, he takes occasion to refer to the fact that the Pythagorean triangle is mentioned in the Sulva-sutras, which he puts as late as the 5th or 4th century B.C. He does not venture, however, upon the question of the antiquity of the theorem in China, a problem which probably can be solved only by the rise of native scholars who can give us a careful textual criticism of the classics of their country. It is interesting to see that Professor Zeuthen, than whom we have few authorities better recognized in the history of Greek mathematics, maintains the common view that the lunes of Hippo-

crates are due to this writer, in spite of some doubt that has recently been cast upon it.

The summary which he gives of the work of the Greeks in geometry is, as would naturally be expected from him, very clear, and forms the principal portion of the work. The section on the early traces of the calculus is especially interesting, and refers of course to the contributions of Democritus, Eudoxus, and Archimedes, but without assigning the credit to Antiphon and Bryson that might be expected.

The mathematics of the Hindus is discussed with brief mention,—too brief, considering the contributions of the writers of India. The reader will not find the well-balanced judgment of the Oriental mathematics, nor the interesting information concerning its algebra, that he would expect from a perusal of the pages devoted to the work of the Greeks. The Arab contributions are somewhat more fully treated.

The medieval period in Europe is considered more at length, this being a period to which Professor Zeuthen has given much attention in his other works. Just at the present time, when Roger Bacon is much in the public eye, it is interesting to note that Professor Zeuthen dismisses him with exactly twelve words, and that these relate solely to optics.

It goes without saying that the book fills its purpose in a satisfactory manner. It gives a popular view of the progress of mathematics down to the period of the Renaissance, and is written in the pleasant style which characterizes all of the works of its distinguished author.

DAVID EUGENE SMITH.

*Geometrie der Zahlen.* Von HERMANN MINKOWSKI. Zweite Lieferung. 1910. viii + 15 pages. B. G. Teubner, Leipzig und Berlin.

THE first volume appeared in 1896. The preparation of the second volume was delayed by unexpected difficulties, and the author published in short articles in the journals most of the results initially intended for the second volume. These articles are accessible in the Gesammelte Abhandlungen of Minkowski, published in 1911 in two volumes, aggregating 872 pages. The present pamphlet is a continuation, bearing the same title, of the closing chapter of the first volume of Geometrie der Zahlen, and contains also a table of contents and index for the two volumes. This highly original contribu-