region at all, and to modify the postulates so as to introduce any points at infinity would be an unpleasant complication resulting in no gain to the geometry. When, however, we desire to map the projective (or inversive) and Lobachevskian planes one upon the other, we find that the Lobachevskian plane lies entirely within a conic of the projective plane (and entirely upon one side of a circle in the inversive plane—there is here no distinction between inside and outside). To perfect the correspondence we adjoin to the Lobachevskian plane the conic (or circle) as an infinite region and the region outside the conic (or upon the other side of the circle) as an ultra-infinite region.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS.,
February, 1914.

FAMOUS PROBLEMS OF GEOMETRY.

"Squaring the Circle," A History of the Problem. By E. W. HOBSON. Cambridge, at the University Press, 1913. iii+57 pp. Price 3s.

A FASCINATING and voluminous volume could be written on ancient problems of geometry, their influence on the progress of mathematics and the various developments in mathematics which contributed to their generalization or final settlement.

There is the familiar problem,* to draw from a given point P a line such that the line segment cut off by two intersecting lines l_1, l_2 shall be of given length. This problem is capable of solution with ruler and compasses in but one case, namely when P is on a bisector of an angle between l_1 and l_2. Suppose this condition to obtain. The problem is not an easy one, in general, but Apollonius (about 225 B.C.), known to his contemporaries as the "great geometer," found an elegant solution.† The complete discussion for the case of l_1 and l_2 at right

* Cf. my paper "Discussion and history of certain geometrical problems of Heraclitus and Apollonius," *Proc. Edinb. Math. Soc.*, vol. 28 (1909–1910), pp. 152–178.

† Although twice proposed in the *American Math. Monthly* (Feb., 1910, vol. 17, p. 48, cf. pp. 140–141; Feb., 1911, vol. 18, p. 44, cf. pp. 114–115) no solution has been forthcoming.

angles, was known to Heraclitus. When P was not on a bisector, the Greeks solved the problem by means of: (*a*) the conchoid of Nicomedes, and (*b*) an hyperbola. More than a thousand years elapsed; then we find Girard (in 1629) dealing with the Heraclitus case and led to an equation of the fourth degree. He gives geometrical interpretation of negative roots, a notable achievement in his time. A few years later (1637) Descartes considered the same problem more generally. Huygens found many solutions, those which are geometrical of the Apollonian case being of remarkable elegance. Newton with keen penetration showed how, with appropriate choice of unknown, the problem leads to the solution of a quadratic equation, whereas, without suitable choice of unknown, we get a reducible biquadratic. Any complete discussion of the problem must also refer to L'Hospital, Ghetaldi, Gergonne, d'Omerique, Steiner, and to the solution of the general problem through methods of projective geometry by G. Russo.

Then recall the "problem of Apollonius," to describe a circle tangent to three coplanar circles, with its many special cases, how it led to developments in geometry.* In 1600 Viète treated the problem by synthetic methods; analytical solutions were found by Newton, Euler and N. Fuss. Fermat discussed, synthetically, the analogous problems for spheres. In later day a solution of this problem was found by pupils of Monge, who discussed the motion of a variable sphere tangent to three given spheres. Dupuis discovered and Hachette proved (1804) that the middle point of the sphere moves on a conic, and that its points of contact describe circles. Soon after (1813) Dupin published his researches on the remarkable surfaces which envelope such a variable sphere, surfaces which he afterward named cyclides. Then followed the ideas of radical axes, power of circles and spheres, similitude, and inversion, connected with developments of Gaultier, Poncelet, Quetelet, Steiner, Plücker, Magnus, Ingram, and Stubbs.

Again, there is the less known problem of Pappus, to inscribe in a circle a triangle whose sides (or sides produced) shall pass through three given points on a line. This is solved by Pappus as a lemma useful for attacking the problem of Apollonius (in just what way is an interesting speculation).†

* Cf. Reye, Synthetische Geometrie der Kugel und linearer Kugelsysteme. Leipzig, 1879.

† Cf. my paper entitled "Centers of similitude of circles and certain theorems attributed to Monge. Were they known to the Greeks?", which is soon to appear in the *Amer. Math. Monthly.*

The generalization of the problem to the case where the given points were any points was found by Simson (1731). After thirty years of attention Castillon gave a solution to the Berlin Academy in 1776. Lagrange, who was present at the meeting, handed a trigonometric solution to Castillon on the next day "apparently for the purpose of showing that an analyst could solve in a single evening a problem that had confessedly required from a geometer of no mean powers many years of study." Then Lexell, Euler, and Fuss discussed the problem but no method was found which could be generalized so that a polygon could be substituted for a triangle. This method was discovered by Giordano di Ottajano, a student at Naples in 1784, when only 16 years of age. Malfatti, Romanus, Lhulier, Carnot, and Meyer Hirsch made later contributions to the discussion. William Wallace, famous for his discovery of Wallace's line (incorrectly called Simson's line), first pointed out (in 1798) indeterminate cases in the triangle problem. To follow up developments in the English school: Lowry made obvious generalization of the problem (in 1806) to a polygon inscribed in an ellipse; Hearn considered the general dual problem of circumscription (instead of inscription) due to Gergonne; and then there are researches of Townsend, Potts, and Renshaw leading up to results of Sir William Rowan Hamilton regarding polygons inscribed in a sphere, an ellipsoid and an hyperboloid, and with sides passing through given points.* But in the early part of the nineteenth century in the French school we find that Gergonne, Servois, Poncelet, Brianchon, and others discussed various phases of the problem of polygons in conics, some of them involving the newly discovered principle of duality.

Similar summary sketches of many other notable problems, with like origin, might be given; among these the most famous are the three problems of the ancients, known under the titles: the duplication of the cube, the trisection of an angle,† and the

* Hamilton considered the solution of these problems which depended on a linear equation in finite differences as an especially tough piece of work. Cf. Life of Sir Wm. R. Hamilton, by R. P. Graves, vol. 3 (1889), pp. 88, 426.

† Who first proved the impossibility of the classic problem of trisection of an angle? I have not met with a statement of this fact in any of the mathematical histories, but surely it was before 1852, when Sir William Rowan Hamilton wrote to De Morgan: "Are you *sure* that it is impossible to trisect the angle by Euclid? I have not to lament a single hour thrown away on the attempt, but fancy that it is rather a tact, a feeling, than a

squaring of the circle. Although all of these are somewhat special in character, they have, as Hobson remarks, "one great advantage for the purposes of historical study, viz. that their complete history as scientific problems lies, in completed form, before us." Taking the last of these problems, Hobson continues, "we possess indications of its origin in remote antiquity, we are able to follow the lines on which the treatment of the problem proceeded and changed from age to age in accordance with the progressive development of general mathematical science, on which it exercised a noticeable reaction. We are able to see how the progress of endeavors towards a solution was affected by the intervention of some of the greatest mathematical thinkers that the world has seen, such as Archimedes, Huygens, Euler, and Hermite. Lastly, we know when and how the resources of modern mathematical science became sufficiently powerful to make possible that resolution of the problem, which, although negative, in that the impossibility of the problem subject to the implied restriction was proved, is far from being a mere negation, in that the true grounds of the impossibility have been set forth with a finality and completeness which is somewhat rare in the history of science."

"If the question be raised, why such an apparently special problem, as that of the quadrature of the circle, is deserving of the sustained interest which has attached to it, and which it still possesses, the answer is only to be found in a scrutiny of the history of the problem, and especially in the closeness of the connection of that history with the general history of mathematical science. It would be difficult to select another special problem, an account of the history of which would afford so good an opportunity of obtaining a glimpse of so many of the main phases of the development of general mathematics; and it is for this reason, even more than on account of the intrinsic interest of the problem, that I have

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proof, which makes us think that the thing cannot be done. No doubt we are influenced by the cubic form of the algebraic equation. But would Gauss's inscription of the regular polygon of seventeen sides have seemed, a century ago, much less an impossible thing, by line and circle?"

De Morgan replied: "As to the trisection of the angle, Gauss's discovery increases my disbelief in its possibility. When $x^{17} - 1$ is separated into quadratic factors, we see how a construction by circles may tell. But, it being granted $ax^3 + bx^2 + cx + d$ is *not* separable into a real quadratic and a linear factor, I cannot imagine how a set of intersections of circles can possibly give no more or less than three distinct points." Graves' Life of Sir Wm. R. Hamilton, vol. 3 (1889), pp. 433–435.

selected it as appropriate for treatment in a short course of lectures."

Thus Professor Hobson gave six lectures at Cambridge University during the Easter term of last year, on the history of the problem of the quadrature of the circle. These lectures now appear in print in the volume before us.

It is 160 years since Montucla published anonymously his little book* on the history of this problem. In 1831, again in anonymous fashion, it was revised and corrected by Lacroix. Some sixty years later came Rudio's elaborate and excellent work.† Then there have been the Bändchen of Schubert‡ and of Beutel.§ Other historical surveys, not published in separate volumes, were written by T. Muir,|| B. Calò,¶ Cantor,** T. Vahlen†† and D. E. Smith.‡‡

Professor Hobson's sketch is divided, as Rudio's, into four chapters: one a general account of the problem, and one for each of three historical periods "marked out by fundamentally distinct differences in respect of method, of immediate aims, and of equipment in the possession of intellectual tools."

Introductory to the general survey we are reminded that from the time of the Greeks down to our own day the problem has been very widely known outside of mathematical circles and many such people have occupied themselves in seeking a solution. In fact, as Schubert remarks, the Greeks had a special word to designate this species of activity, namely *τετραγωνίζειν*, which means to busy one's self with the quad-

* Histoire des recherches sur la Quadrature du cercle . . . avec une addition concernant les problemes de la duplication du cube et de la trisection de l'angle, Paris, 1754, pp. xliii+304+8 pls.

† Archimedes, Huygens, Lambert, Legendre. Vier Abhandlungen über Kreismessung . . . mit einer Übersicht über die Geschichte des Problemes von der Quadratur des Zirkels, Leipzig, 1892, pp. viii+166.

‡ H. Schubert, Die Quadratur des Kreises in berufenen und unberufenen Köpfen. In Virchow-Holtzendorffs Sammlung, Heft 67, Hamburg, 1889. Translated in *Monist*, Jan., 1891, vol. 1, pp. 197–228; reprinted in *Smithsonian Report* for 1890, pp. 97–120.

§ E. Beutel, Die Quadratur des Kreises (Mathematische Bibliothek, Nr. 12), Leipzig und Berlin, 1913, pp. 75.

|| Encyclopaedia Britannica, ninth edition, article "Squaring of the circle," 1887; eleventh edition, article "Circle," 1910.

¶ In Enriques's Fragen derElementargeometrie, 2. Teil, Leipzig u. Berlin, 1911.

** Vorlesungen über Geschichte der Mathematik, vol. 1, 3. Aufl., 1907; vol. 2, 2. Aufl., 1900, reprinted 1913; vol. 3, 2. Aufl., 1901; vol. 4, 1908.

†† Konstruktionen und Approximationen, Leipzig u. Berlin, 1911.

‡‡ In Monographs on Topics of Modern Mathematics, ed. by J. W. A. Young, New York, 1911.

rature. In later day, among the scientific cranks with whom almost every scientific society has had to deal, is the insistent circle squarer amusingly dissected by De Morgan in his Budget of Paradoxes. So pestered were the members of the French Academy that as far back as 1775 they passed a resolution not to examine, from that time on, any so-called solutions of the quadrature of the circle that might be handed in.

On pages 4–10 Hobson sets forth the implications of the problem and notes that the fact was well known to the Greeks that the problems of the quadrature and rectification of the circle were really equivalent. He emphasizes assumptions and limitations of euclidean constructions and remarks, without direct reference to Mascheroni, that all such are possible with compasses alone. His discussion is then summed up: As an ideal problem the quadrature of the circle "is capable of accuracy bounded only by the instruments employed. Ideally we can actually determine, by euclidean methods, a square of which the area differs from that of a given circle by less than an arbitrarily prescribed magnitude, although we cannot pass to the limit."

The first historical period commences with the early empirical determinations of the ratio of the circumference of a circle to its diameter, and ends with the invention of the calculus in the latter part of the seventeenth century. This period was characterized by geometric discussion, for example, the method of exhaustions coupled with polygons circumscribed and inscribed. The first determination of the value of π is to be found in the Papyrus Rhind preserved in fragmentary condition in the British Museum and now being translated into English by Chancellor and Mrs. Chace of Brown University. This papyrus, written about 1700 B.C., is based upon a much earlier work. We here find $\pi = 256/81 = 3.1604 \cdots$. From the time of Chou-Kong, who lived in the twelfth century B.C., Chinese mathematicians employed the approximation $\pi = 3$. This value was also known to the Babylonians and is implied in verses of the Bible in Kings and Chronicles which are believed to have been written in the sixth and fifth centuries B.C. About 420 B.C. Hippias of Elis invented, for the solution of the problem, a curve known as the τετραγωνίζουσα or quadratrix, and about the same time Hippocrates of Chios developed his theory of menisci or lunulæ. But the first really scientific treatment was given by Archimedes

(287–212 B.C.). Considering circumscribed and inscribed polygons of 96 sides, he was led to the result $3\frac{1}{7} < \pi < 3\frac{10}{71}$ that is $3.14285 \cdots < \pi < 3.14084 \cdots$. The next more exact determination was found by Ptolemy (87–165 A.D.) to be $3\frac{17}{120} = 3.14166 \cdots$. Four centuries later the great Chinese astronomer Tsu Ch'ung-chih proved that 10π lies between 31.415927 and 31.415926 and thence deduced the value $\frac{355}{113}$,* which is correct to six places of decimals. François Viète (1540–1603) obtained the value of π correct to 9 places, Adrianus Romanus to 15, Ludolph van Ceulen to 35. Huygens, Descartes, and Gregory introduced new methods of discussing the problem. Of approximate euclidean solutions may be mentioned that of J. de Gellert (*Grunert's Archiv*, 1849) which is readily deduced from the fact that $\dfrac{355}{113} = 3 + \dfrac{4^2}{7^2+8^2}$. The integraph, invented and described by the Russian engineer, Abdank-Abakanowicz, and its application to the construction of π, are nowhere mentioned by Hobson.

The second period, from the second half of the seventeenth to the latter part of the eighteenth century, is characterized by analytic discussion of the problem. J. Wallis, Lord Brouncker, Gregory, Machin, Leibnitz, Euler, and others obtained various expressions for π involving inverse tangents of numerical quantities, definite integrals, infinite products and series. Machin's relation

$$\frac{\pi}{4} = 4 \tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$$

was used by Shanks who gave, in 1873, the value of π correct to 707 places of decimals. Professor Hobson dwells on Euler's work, for it is recalled that the final settlement of the problem was based upon the remarkable relation which he discovered,† namely

$$e^{i\pi} + 1 = 0.$$

* This is very easily remembered by writing down the first three odd numbers, each repeated as 113355, and recalling that the first three digits are to be placed in the denominator, the last three in the numerator.

† The form $e^{ix} = \cos x + i \sin x$ was first given by Euler in *Miscellanea Berolinensia*, vol. 7 (1743), p. 179; paper read Sept. 6, 1742. He used the symbol e in a letter dated November 25, 1731 (Corresp. Mat. et Phys., par Fuss, vol. 1 (1843), p. 58. He employed i for $\sqrt{-1}$ in 1777 (Institutionum calculi integralis, vol. 4 (1794), p. 184: "De formulis differentialibus . . . ," presented to the Akademie, May 5, 1777). It should be noted

Although in this period great progress was made toward the solution of the problem, the true nature of the number π had not yet been discussed; but from the end of the seventeenth century none of the prominent mathematicians doubted that π was irrational.*

The third period commences with the proof of this fact by J. H. Lambert (1728–1777). Rudio's statement that this proof "fehlte zur völligen Strenge" has been copied without checking by Klein, Smith, and others, but Pringsheim showed that Rudio was in error. Following him, Vahlen and Hobson state the matter correctly. Nearly 80 years were to elapse before Liouville proved the existence of transcendental numbers. Hobson reproduces the 1851 presentation, in the main. Not till 1873 did Hermite prove the transcendence of e. Finally in 1882 Lindemann succeeded in establishing the transcendence of π and the problem whose history covers 4000 years was finally solved. Simplifications of Lindemann's proof soon followed, and one of these by Gordan is the basis of the condensed four-page Hobson proof, which requires knowledge of the elements of the differential calculus. It is assumed on page 55 that there are an infinite number of prime numbers. This was known to Euclid (Elements XI, 20.)

On page 50 the author establishes the theorem: "In order that a point P may be determinable by euclidean procedure it is necessary and sufficient that each of its coordinates be a root of an equation of some degree, a power of 2, of which the coefficients are rational functions of $(a_1, a_2, \cdots, a_{2r})$, the coordinates of the points given in the data of the problem." Hobson starts with

$$x = a + b\sqrt{c_1 \pm \sqrt{c_2 \pm \sqrt{c_3 + \cdots}}} + b'\sqrt{c_1' \pm \sqrt{c_2' \pm \sqrt{c_3' + \cdots}}} + \cdots \tag{1}$$

But it is not shown that the equation referred to in the theorem may not have other roots than those given by (1). Multiple

that Roger Cotes gave the formula $ix = \log(\cos x + i \sin x)$ (*Phil. Trans.*, 1714), which is practically the same as the above as we look at it now, long before Euler.

* Euler, Introductio in Analysin Infinitorum, vol. 1 (1748), p. 93. It would be interesting to have Professor Hobson's authority for the statement that Euler gave expression to the conviction "that e and π are not roots of algebraic equations."

roots derived from (1) are not considered either, and the term "irreducible equation" is not mentioned. For this discussion as well as for the proof of the transcendence of π, the student will probably find more satisfactory and more elementary treatment in Klein's remarkable little book, Famous Problems in Elementary Geometry,* to which our author nowhere refers.

Professor Hobson's most interesting and attractively written book is such as to fire the imagination of a young student of mathematics and as to impel him to more persistent and enthusiastic effort. It is sure to have a wide circulation. So much the more then is it to be regretted that lamentable lack of care is displayed in the proofreading and final checking of details, by the author. His authorities, we are informed, included Heath,† Rudio, Cantor, Mikami,‡ and Vahlen, who can rarely, if ever, be blamed for the errors committed. Against an historical work coming from such a source, these are serious charges. It may therefore be well to support them at some length.

Obvious slips in typography occur on page 46, lines 3 and 10, page 55, line 18, in the exponent of C. On page 55, lines 25, 27, 30, with usual notation for differentials, brackets should surround the exponents of ϕ. In mathematical writing, uniformity seems highly desirable, at least within the limits of a 60 page book: why then the proper names in italics on pages 23, 25, 27, and why the three forms Leonardo Pisano, Nicholas of Cusa, Leonardo da Vinci? Three different methods are used for referring to the volume of a periodical work (for example, page 31, and page 44, lines 6 and 9) while one of these methods is elsewhere used for a book in a volume (page 31, line 29).

Before the abbreviation "Kl." on page 44 and again in a footnote on page 53, "Math.-nat." should be placed. Many names are spelled incorrectly: In seven places an extra *h* is put in Huygens, Raaus should be Ra-ā-us', for Aryabhatta read Âryabhaṭṭa; for Chang Hîng, Chang Hêng; for Wang Fau, Wang Fan; for Gancea, Ganeca; for Alchwarizmi, Alchwarizmî; for George Purbach, Georg von Peurbach

* Boston, 1897; German ed., Leipzig, 1895; French ed., Paris, 1896; Italian ed., Torino, 1896; Russian ed., Kasan, 1898.

† Works of Archimedes, Cambridge, 1897.

‡ The Development of Mathematics in China and Japan, Leipzig, 1912.

or Georg of Peurbach; for Rheticus, Rhäticus; for Ludolf van Ceulen, Ludolph van Ceulen; for Willebrod Snellius, Willebrord Snellius; for Grünert, Grunert; for Tanyem Shōkei, Tanzan Shōkei.

We are told that Professor Landau discussed lunulæ in 1890, that is when he was 13 years of age. This date should be 1902, and the reference to "Ueber quadrierbare Kreisbogenzweiecke," *Sitzungsberichte der Berliner Mathematischen Gesellschaft,* volume 2 (1902), pages 1–6. Three dates of birth are given incorrectly: Ludolph van Ceulen, 1539 instead of 1540 (N.S.); Newton, 1642 instead of 1643 (N.S.); Snellius, 1580 instead of 1581. The phrase "is said to have," page 27, line 4, should be deleted and after the next sentence, "According to his wish the value was engraved on his tombstone which has been lost," should come: "but the inscription has been preserved (Bierens de Haan, *Messenger of Mathematics,* volume 3 (1874), page 25)."

Vega calculated π correctly to 136 not 140 places.* Two slips occur in connection with W. Shanks' name (page 39): (1) he published his value of π to 530 places in *Proceedings of the Royal Society,* 1853–54, but his value to 607 places in Contributions to Mathematics, London, 1853, pages 86–87; (2) he gave 707 places in 1873, not "1873–74." On page 40 for "1766" read "1769," the year when the result was printed; and on page 39 for "1755" read "1779 (though not published till 1798);"† for "The same series was also discovered independently by Ch. Hutton" read "The same series was first discovered by Ch. Hutton."

The statement on page 41: "He [Euler] introduced the practice of denoting each of the sides and angles of a triangle by a single letter," is inaccurate, as Caswell‡ (and Oughtred) had already named each of the sides by a letter. Hobson might have correctly written: "Euler was one of the first if not the first to introduce the notation a, b, c, for the sides

* G. Vega, Thesaurus Logarithmorum, Leipzig, 1794, p. 633, and W. Shanks, Contributions to Mathematics, London, 1853, p. 86.

† "Conventuri exhibita die 7 Junii, 1779" *Nova Acta Acad. sc. Petrop.,* vol. 11 (1793), 1798, p. 133. Cf. G. Eneström, Verzeichnis der Schriften Leonhard Eulers, Leipzig, 1910, p. 181.

‡ "Suppose m, n the legs of the angle required; B its base . . . $\zeta = \frac{1}{2}$ sum m, n, B," page 6 of A Brief (but full) Account of the Doctrine of Trigonometry Both Plane and Spherical, by John Caswell, London, 1685; bound in with Wallis's Treatise of Algebra, London, 1685.

opposite the angles A, B, C, of a triangle, etc."* And again, did Euler introduce "the short designation of the trigonometrical ratios by $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, etc."? The originals of the remarkable memoirs of 1753 and 1781† are not available to me, but in E. Hammer's translations‡ "tan" never occurs; it is always "tg"; so also "ctg" not "cot"; furthermore the abbreviation "cosec" or "csc" does not come in at all. In 1777 J. T. Mayer used the following abbreviations:§ sin, cos, tang, cot, sec, cosec.

Professor Hobson recalls that "the notation π appears as early as 1706, when it was used by William Jones in Synopsis palmariorum matheseos." It would have been of interest to have further remarked that the notation was probably suggested to Jones by Oughtred||, whose name stands out much more prominently in English mathematics. Liouville's papers on transcendental numbers are dated 1844¶ and 1851,** not 1840 (page 44). To the sentence: "The simpler of Liouville's methods of proving the existence of such numbers will be given here" should be added, "with some alterations." And finally Professor Hobson seems to have forgotten (page 53) that it was the third, not the second, edition of his Treatise on Trigonometry which was published in 1911.

In broad outline the work is accurate; but when the mathematician writes now-a-days, he is expected to have a much higher ideal. As clearly stated in the preface, no original contribution to the history of the subject is attempted; for this we must await the forthcoming edition of Pappus by

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* This was done in the case of *spherical* triangles in 1753: "Principes de la trigonométrie sphérique tirés de la méthode des plus grands et plus petits," *Mém. de l'Acad. d. Sc. de Berlin*, vol. 9 (1753), 1755, pp. 223–257.

† *Acta Acad. Sc. Petrop.*, 1779; I, printed 1782, pp. 72–86. "Nach den Akten am 12 März, 1781 der Petersburg Akademie vorgelegt." (G. Eneström, Verzeichnis der Schriften L. Eulers, Leipzig, 1910, p. 149.)

‡ Ostwald's Klassiker der exakten Wissenschaften, No. 73, Leipzig, 1896.

§ Gründlicher und ausführlicher Unterricht zur praktischen Geometrie, Erster Theil, Göttingen, 1777, p. 5 ff.

|| William Oughtred (1574–1660) in his Clavis Mathematica of 1631, etc., and in his Theorematum in libris Archimedes de Sphaera et Cylindro Declaratio, Oxford, 1652, frequently employs the symbol $\delta:\pi$ or $\pi:\delta$ (in modern rotation) for the ratio of the semi-diameter to the semi-periphery or of semi-periphery to semi-diameter. It is noticeable that these letters are *never* used separately, that is, π is *not* used for "Semiperipheria," as Tropfke suggests (Geschichte der Elementar-Mathematik, vol. 2 (1903), p. 135). Oughtred states specifically in his "Theorematum": "$\pi R/\delta$, est semiperipheria circuli cujus Radius est R."

¶ *Comptes Rendus*, vol. 18, pp. 883, 910.

** *Liouville's Journal*, vol. 16, pp. 134, 139.

Professor Hobson's fellow countryman, Sir Thomas L. Heath. It is certainly not well to overload a work of this kind and suggestions for filling up the many lacunae are hardly in order. It appears to the reviewer, however, that the value and usefulness of the work would be considerably enhanced if many more exact references were given to original sources as authorities for the statements that are made, and if an index of names were appended.

In conclusion, and in illustration of the statement made above, that others beside mathematicians were cognizant of the problem of squaring the circle, it may be of interest to put on record two references to literary classics. In the Birds (produced in 414 B.C.) of Aristophanes there is a long dialogue between Peisthetaeros and Meton, the great astronomer, introduced as a solemn quack talking unintelligible nonsense for the most part. In lines 1004–5, however, Meton is made to say:*

> "With the straight rod I measure out, that so
> The circle may be squared."

The second reference takes us to the early fourteenth century. It is to the last canto of the Paradiso where Dante compares his inability to penetrate by his own unaided power the mystery of the Incarnation, to that of one

> "Who versed in geometric lore, would fain
> Measure the circle;† and though pondering long
> And deeply, that beginning, which he needs,
> Finds not; . . . ."‡

Are there references to other passages of this kind before 1500? One may turn to the great Oxford Dictionary for those of later date.

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PROVIDENCE, R. I.,
January 30, 1914.

* Translation of B. B. Rogers, London, 1906.
† "Misurar lo cerchio;" Longfellow translates this "To square the circle."
‡ Cary's translation, Canto 33, lines 133–135.