years. Therefore in this notice it will be necessary to indicate only the changes (not many in number) which have been made in the second edition. These are of two kinds: changes in content, and changes in arrangement and printing.

The principal changes in content consist of two additions. There is an introduction taken from the chapter entitled Le hasard in Poincaré's Science et Méthode. It has to do with the philosophical considerations connected with the possibility of a mathematical theory of probability. There is a fresh chapter at the close of the book dealing with a number of miscellaneous questions. Besides this there are some rearrangements of old matter and additions of new matter throughout the book; but in no cases do these changes seem to be of sufficient importance to require separate consideration.

In the first edition the material was grouped by lectures and not by topics, and no page headings were given to indicate the nature of the contents at any place. On this account the book was inconvenient for purposes of reference. In the second edition there is an arrangement of the matter by topics into chapters and page headings are given to indicate the chapter to which any page belongs. This adds greatly to the reader's comfort and will increase the usefulness of the book.

Concerning a work of Poincaré's, one scarcely needs to add that it is interesting and valuable to the student of the subject with which it deals.

R. D. CARMICHAEL.

The Dynamical Theory of Sound. By HORACE LAMB. New York, Longmans, Green and Co. (London, Edward Arnold), 1910. viii + 303 pp.

To the two hundred or more foreign mathematicians gathered at Cambridge last summer the atmosphere of the Congress may well have appeared somewhat foreign; for prominent among the "home talent" were Sir George Darwin, the president, Lord Rayleigh, the honorary president, Sir J. J. Thomson, Sir Joseph Larmor, M.P., and Professor E. W. Brown, three of the lecturers, all in the front rank of mathematicians in the Cambridge sense, but elsewhere ranked rather as physicists or astronomers. Indeed, although Cambridge has been and still is graced by the presence of eminent pure mathematicians, there is no more striking phenomenon in university history, no more persistent and justified tradition, than the preeminence of Cambridge as *the* school of the dynamical explanation of the universe, foremost from Newton to Maxwell and on to the present day. Among the treatises, standards for the world, and issue of this school, are Lord Rayleigh's Sound and Lamb's Hydrodynamics; and now Lamb offers us out of his mature experience an elementary Sound, a sort of lesser Rayleigh, written in a delightful style and valuable both for itself and as an introduction to the greater work.

Recently we have had occasion to remark that geometrical optics has fallen between the mathematician and physicist into the hands of the optical engineer.* If we may judge by the pamphlets circulated by our university departments, sound has likewise, and probably for similar reasons, been abandoned by mathematician and physicist. Indeed one such pamphlet makes bold to state that sound is not the subject of any course in physics, but in some of its important aspects is treated in a certain course in mathematics. Needless to say, this course is on harmonic analysis and the aspects of the theory of sound therein treated are merely those connected with the determination of solutions of certain differential equations subject to particular boundary conditions. These problems are adequately treated by Lamb, not only from the mathematical, but from the dynamical and physical viewpoints. There are, however, numerous other phenomena of sound, sometimes connected with common and simple physical instruments, which the author discusses, and which he must sketch more from the physical side, less from the mathematical, because the complete mathematical solution offers too great difficulties in analysis.† Thus a considerable part of the work assumes a semi-descriptive tone.

Mathematicians are offered a number of interesting problems in the theory of sound somewhat more advanced than elementary harmonic analysis. For example, there is the theory of finite waves, where the differential equations are no longer linear. Riemann, Hugoniot, Hadamard ‡ is the sequence of names which should be mentioned in this connection. Lamb merely discusses the matter briefly with a reference to

* This BULLETIN, November, 1912, p. 74.

† The megaphone, for instance, is a simple object with familiar effects, but its mathematical theory is not by any means simple.

‡ See a review of Hadamard's Théorie des Ondes, this BULLETIN, vol. 10, pp. 305–317.

Riemann. Perhaps, however, the topic now most likely to draw mathematicians back to the abandoned theory of sound is the theory of integral equations and its applications to the integration of differential equations of physics. But unfortunately for the practical man the series which arise in Fredholm's solutions, though very rapidly convergent, are extremely difficult to compute, owing to the complexity of each term. It may be, therefore, that for special problems apart from existence theorems we may still be forced to use something more akin to harmonic analysis, perhaps the method of Ritz.* These matters the author very properly omits from this elementary treatise.

As for matters of detail, the book, after a short introduction, takes up the study of vibrations. The pace is moderate, passing successively the simple pendulum, the general system of one degree of freedom, forced vibrations, resonance, friction and damping, systems of several degrees of freedom, and the transition to continuous systems. The principle that the periods of the normal modes are "stationary" is not overlooked,—nor is the principle of reciprocity. Although both these principles are mathematical, they are unfortunately omitted from most treatises which are not primarily interested in the physical foundation of the subject. Chapter II is on the vibrations of strings, and leads to the study of Fourier's theorem (Chapter III). This is followed by a chapter on the vibrations of bars, and one on membranes and plates. These developments have filled about one half of the volume. Apart from the derivations of the differential equations, the considerations of energy, and the discussions of the various limitations which actual physical conditions may impose, the work is such as might well be found in a mathematical course on harmonic analysis.

In passing it should be observed that the page of this book has a very pleasing and restful appearance; it is neither too large nor too small, too open not too closely packed. The typography, though excellent, could be improved in two particulars: 1°, by the use of mortised integral signs, wherever a lower limit has to be set; 2°, by raising the periods and commas when occurring immediately after the central line of

* Reference may be made to the illuminating developments and comments of Poincaré, Leçons de Mécanique céleste, tome 3: Théorie des Marées, chap. 10.

a fraction. As this latter improvement, however, is unfamiliar in our BULLETIN and *Transactions*,* we regret throwing the stone!

In the sixth chapter, after discussing the elasticity of gases, the velocity and energy of plane sound waves, reflection, and the vibrations of a column of air, Lamb takes up waves of finite amplitude and the possibility of the propagation of a wave of discontinuity. He gets far enough to run into the difficulty, first signalized by Lord Rayleigh, in regard to the conservation of energy; and then, instead of discussing Hugoniot's law of dynamic adiabaticity, he remarks that obviously no complete theory of waves of discontinuity can be attempted without some reference to viscosity and thermal conduction, and he therefore proceeds to treat these two subjects. His remark is both right and wrong; it is wrong in stating that no complete theory of waves of discontinuity can be attempted without some reference to viscosity, for this is precisely what Hugoniot and his follower Hadamard have attempted and accomplished with results well in accord with Vieille's experiments; it is right because there is no â priori reason for disregarding viscosity, especially in the case of waves of discontinuity. The chapter ends with the treatment of the damping of waves in narrow tubes and crevices.

The author is now ready for the general theory of sound waves (Chapter VII), the general equations of fluid motion, specialized for sound, spherical waves, waves resulting from a given initial disturbance, point sources, reflection, refraction due to temperature gradients, and refraction by wind. Chapter VIII continues with spherical waves and point sources, and applies them to the waves given off by a vibrating sphere or other solids. The scattering of sound by an obstacle and its transmission through an aperture (diffraction) are briefly discussed. Only the theory of pipes and resonators (Chapter IX) and some account of physiological acoustics (Chapter X) is needed to round out and complete the work.

It should be evident that the author has exhibited excellent taste and balance in his selection and treatment of topics, that he has accomplished just what he intended, and that it was worth accomplishing,—a text that by easy stages fits the reader for the more advanced treatises and, we may add, a

* So far as we recall the only people who know how to set commas after fractions are Ginn and Co. and Gauthier-Villars; others drop them too low so that they either look lost or look like accents to the denominator.

text that by its graceful composition can hardly fail to lure the reader on to the further study of the subject. All this would have been predicted in advance of reading the Sound by anybody at all familiar with the author's Hydrodynamics.

EDWIN BIDWELL WILSON.

NOTES.

THE opening (January number) of volume 14 of the *Transactions of the American Mathematical Society* contains the following papers: "The triad systems of thirteen letters," by F. N. COLE; "Triple system sas transformations, and their paths among triads," by H. S. WHITE; "Proof of Poincaré's geometric theorem," by G. D. BIRKHOFF; "On the existence of loci with given singularities," by S. LEFSCHETZ; "Singular multiple integrals, with applications to series," by B. H. CAMP; "Decomposition of an n-space by a polyhedron," by OSWALD VEBLEN; "On convergence factors in double series and the double Fourier series," by C. N. MOORE; "Algebraic surfaces invariant under an infinite discontinuous group of birational transformations. Second paper," by VIRGIL SNYDER; "Note on Van Vleck's non-measurable sets," by N. J. LENNES; "Some asymptotic expressions in the theory of numbers," by T. H. GRONWALL; "Determination of the finite quaternary linear groups," by H. H. MITCHELL; "On the character of a transformation in the neighborhood of a point where its Jacobian vanishes," by L. S. DEDERICK.

THE December number (volume 14, number 2) of the *Annals of Mathematics* contains the following papers: "Two theorems on conics," by S. LEFSCHETZ; "A new type of solution of Laplace's equation," by H. BATEMAN; "Involutoric circular transformations as a particular case of the Steinerian transformation and their invariant nets of cubics," by A. EMCH; "On analytic functions of constant modulus on a given contour," by T. H. GRONWALL; "Necessary and sufficient conditions for the interchange of limit and summation in the case of sequences of infinite series of a certain type," by T. H. HILDEBRANDT; "A simple proof of a fundamental theorem in the theory of integral equations," by MAXIME BÔCHER; "An application of modular equations in analysis situs," by O. VEBLEN.