elliptic partial differential equation, the author includes a good introduction to the integral equation according to Fredholm. The chapter is clear and not too brief for the reader for whom the book is intended, though the following chapter on boundary problems in ordinary linear differential equations of the second order might have been shortened with profit in a work of this type and title. One quarter of the book is devoted to the introduction to integral equations. In chapter seven the results of the two previous chapters are applied to particular equations of the elliptic type. Properties of the solutions of

$$\Delta u = 0,$$

in particular two boundary value problems, are treated at some length, following Fredholm and Hilbert in treatment and notation. A few pages are devoted to special points connected with the solutions of

$$\Delta u + 2\pi\varphi(x, y) = 0, \quad \Delta u + \lambda u = 0,$$
$$\Delta u + \lambda k(x, y)u = 0 \; (k > 0).$$

The volume closes with a short chapter on some partial differential equations of physics.

The book is clear and logical. Generalities are illustrated by well-chosen special examples. After deciding upon the content the author keeps to the point and does not forget the student for whom he is writing. As to the content, of course there will be differences of opinion as to the choice of topics from such a wide field. For example it would not have been a difficult task to give some notion of Lie's methods without an increase in size. This volume is well worthy of a place in a series which includes Schlesinger's little work.

A. R. CRATHORNE.

Lehrbuch der Differentialgleichungen. Von A. R. FORSYTH. (Mit den Auflösungen der Aufgaben von HERMANN MASER.) Zweite autorisierte Auflage, nach der dritten des englischen Originals besorgt und mit einem Anhang von Zusätzen versehen von WALTHER JACOBSTHAL. Braunschweig, Vieweg und Sohn, 1912. xxii + 921 pp.

FOR many years Forsyth's Treatise on Differential Equations has held a place of importance among physicists and other workers who have much to do with the practical problem of solving differential equations. The value of the book in this respect is due to its fullness of practical methods of integration and its great number of well-chosen examples. The purpose of the second German edition is to extend the range of subjects beyond those embraced in the third English edition so as to make the book more useful to students of pure mathematics. Nevertheless no use is made of function-theoretic considerations; and the discussion is confined almost entirely to real variables.

For convenience of review the book may be divided into three parts. The first (pages 1–526) contains a translation of the third English edition; the second (pages 527–664) contains the new matter supplied to the German edition by the translator; while the third (pages 665–912) contains solutions of exercises. There is added an author index and a subject index.

The third part of the book contains Maser's solutions of the exercises found in the second English and in the first German edition, with numerous corrections made by the translator. No solutions of the additional problems contained in the third English and in the present German edition are given.

The first part of the book follows the third English edition. Throughout the treatment of ordinary equations there are many footnotes referring to the discussion given in the Zusätze in the second part, where additions to and sometimes criticisms of the statements in the text are to be found. No such additional remarks are adjoined to the treatment of partial equations. In at least one important case it seems desirable that such should have been done. The classification of integrals of partial differential equations (§§ 177–183) is not satisfactory. The classification is arrived at by means of a formal process of elimination and there is no inquiry as to the range of validity of the formal process. The resulting classification of integrals into three kinds is incomplete; there are integrals which do not come in any of these classes. For a discussion of this matter see Forsyth's Theory of Differential Equations, volume V, chapter V.

There remains yet for consideration the part of the book which contains new matter; and of this we shall speak in more detail. Only the principal additions will be considered, numerous shorter notes not being mentioned at all. In this part attention is confined entirely to ordinary equations.

Existence theorems are given (pages 529–541) for a single equation of the first order, for n equations of the first order, and for a single equation of the nth order. The argument is presented in such form as to be intelligible to one who has no further acquaintance with functions than is obtained in a good course on integral calculus. The method is that of Cauchy and Lipschitz. Thus we have a satisfactory treatment for a student's first introduction to the rigor of an existence theorem.

The short discussion of algebraic solutions and addition theorems (pages 545–550) cannot fail to be interesting and instructive to the student. The systematic treatment of the theory of integrating factors (pages 551–556) is also a welcome addition.

The general discussion of singular solutions given in the English edition (and reproduced in the present one) is somewhat too difficult for the beginner. Consequently, a more elementary treatment of this subject is given (pages 556–565). There can be no doubt that this addition is a desirable one.

An interesting treatment of the formal problem of integrating by series is given on pages 573–597. The discussion is limited to linear homogeneous equations. The only problem considered is that of the formal determination of power series which formally satisfy the equation. All convergence proofs are omitted. The general problem of finding the power series expansions which formally satisfy the equation is worked out in a way which is very satisfactory from the point of view of the beginner. The treatment is not entirely complete, some exceptional cases being omitted, as for instance, when the roots of the indicial equation differ by an integer. But when the equation is of the second order the treatment is made complete.

The important particular case when the recursion formula for determination of the coefficients of the series expansion has only two terms is given a special detailed treatment (pages 589–597). The importance of this case, especially in a student's early study of integration by series and in applications to equations of physics, will make this section a welcome one to many teachers. The matter is well presented and the results are stated in such form as to be easy of reference (as is indeed the case with all the more important results contained in the Zusätze).

The application (pages 597–617) of the theory of integration by series to the Legendre, Bessel, and hypergeometric equations will be useful to the student.

The last of the important Zusätze (see pages 637–663) is devoted to an exposition of the theory of systems of simultaneous differential equations of the first order.

Having thus passed in quick review the contents of the book, it is now apparent that the translator has certainly accomplished his purpose of making it more useful to the student of pure mathematics. There remains the question whether there are not other additions which would have been desirable in accomplishing this purpose. There is at least one which, in the reviewer's opinion, should have been inserted.

In connection with the theory of formal integration by series it would have been easy to insert a proof of the convergence in general of the series for the case of second order equations; and one can but regret that this was not done. Such a treatment would have required only a half-dozen pages; and it would have added greatly to the value of this already valuable section. The translator's reason for omitting it is obvious; he was making no use of function-theoretic considerations, and such a proof would have required the introduction of these notions. But the great value to the student of having at hand this proof, for the relatively simple case of second order equations, seems to be more than a justification for departing in this case from the general plan of the book; it seems indeed to be a demand for it.

On the whole the translator has rendered a distinct service to beginners in the modern theory of differential equations. The Zusätze which he has inserted in this volume have to do with well-selected topics and the treatment is for the most part very satisfactory. The careful arrangement of material and the numerous and convenient cross-references given throughout the Zusätze and the Auflösungen are especially to be commended as contributing to the reader's comfort. The usefulness of such mechanical conveniences is often overlooked by authors.

R. D. CARMICHAEL.

Calcul des Probabilités. Par H. POINCARÉ. Rédaction de A. QUIQUET. Deuxième edition, revue et augmentée. Paris, Gauthier-Villars, 1912. 335 pp.

THE first edition of Poincaré's Calcul des Probabilités was published in 1896; thus it has been before the public for sixteen