The majority of the papers of this third volume are short notes of a few pages each, and deal for the most part with algebra, the integral calculus, theory of numbers, differential equations, and elliptic functions. Three of these are extracts from Hermite's Cours d'Analyse; another is from his autographed lectures at the Ecole Polytechnique.

By far the most important paper is the celebrated "Applications des Fonctions elliptiques" appearing first in the *Comptes Rendus,* beginning in 1877, and afterwards in book form in 1885. This memoir occupies more than 150 pages of the present volume, and contains Hermite's epoch making researches on Lamé's differential equation.

The present volume brings another paper of less real importance but of far more sensational nature, namely the one on "La Fonction Exponentielle." Here Hermite shows that e, the base of the Naperian logarithms, is indeed a transcendental irrationality. A cruel fate robbed him of the glory of proving that π is also transcendental, and yet it would have been but a short step for him to make. In a letter to Borchardt he writes: "Je ne me hasarderai point à la recherche d'une démonstration de la transcendence du nombre π. Que d'autres tentent l'entreprise, nul ne sera plus heureux que moi de leur succès, mais croyez-m'en, mon cher ami, il ne laissera pas que de leur en côuter quelques efforts." Hermite had not long to wait, for nine years later, in 1882, Lindemann brought the long sought proof, and so established the impossibility of "squaring the circle."

The volume is graced with a portrait of Hermite, at about sixty-five. It is a striking likeness; but the kindly look about the eyes will be missed by those who knew him.

JAMES PIERPONT.

Naturwissenschaften und Mathematik im klassischen Altertum. Von J. L. HEIBERG, in Kopenhagen. Mit 2 Figuren im Text. Teubner, Leipzig, 1912. 102 pp. M. 1.25.

IN our generation there have been three men who were preeminently fitted both by taste and by training to write upon the mathematics of the classical civilization. Others have been able to undertake the task in a satisfactory manner, as witness the labors of scholars like Zeuthen, Loria, and Moritz Cantor, but there always stand out three names of men whose love for Greek science and perfect command of the classical languages fitted them in a remarkable manner for investigation in this field. It is unnecessary to name these men to anyone who has worked in the history of mathematics, but it may be permitted, because of other readers, to mention the work of the late M. Paul Tannery, the great contributions of Sir Thomas Heath, and the noteworthy editions of the Greek classics in mathematics that have appeared from time to time under the editorship of Professor Heiberg.

It is therefore particularly fortunate that the publishers of "Aus Natur und Geisteswelt" were able to secure the services of Dr. Heiberg in the preparation of this little work. No one could speak with greater authority upon the subject, and few could so successfully condense the important facts for the general reader. Taken in connection with Professor Zeuthen's summary of the history of mathematics, now in proofsheets and soon to appear in Die Kultur der Gegenwart, the student of the subject will have two excellent points of departure for serious work.

Dr. Heiberg begins, as would of course be expected, with the natural philosophy of the Ionian school, covering a period in which mathematics, physics, and cosmography were closely allied. He then considers the Pythagorean movement, particularly with reference to astronomy, geometry, and the theory of numbers. A chapter is then assigned to medical science as it developed in the fifth century B. C., particularly under the influence of Hippocrates. The development of mathematics in the same period, from the time of Pythagoras to that of Plato, is then discussed. Chapters V and VI deal with the labors of Plato and of Aristotle, respectively, together with those of their followers. These are followed by the longest chapter in the work, one devoted to the Alexandrian period, in many respects the most important of all antiquity. Chapter VIII is happily entitled "Die Epigonenzeit," a period extending through the second and first centuries B. C. In this period we meet the names of Asclepiades of Bythnia in medicine, Alexandros of Myndos in zoology, Theodosius in the study of the sphere, Diocles in geometry, and various others who may properly be designated as epigones.

The last two chapters relate to the feeble contributions of Rome and to the Greek scientific literature of the Empire and the Byzantine period.

It is hardly necessary to remark that such a brief presentation of the subject cannot be critical in its nature. To some extent this defect is remedied by the brief bibliography at the end, and by a list of source material.

DAVID EUGENE SMITH.

Methodologisches und Philosophisches zur Elementar-Mathematik. Von G. MANNOURY. Haarlem, P. Visser Azn., 1909. 276 pp.

THERE appear from time to time, and in various countries, works of more or less merit that relate to the border line or the neutral ground between mathematics and philosophy, not attempting to eradicate existing boundaries, but seeking to show the relations that continually appear when one considers the two regions. We find the same thing on the other side of mathematics, where it borders upon the various physical sciences, and at the present time this region is particularly in the educational limelight. From the standpoint of the lover of pure science the former domain is the more interesting and important, while to him whose interests are chiefly in the utilities the latter has more significance.

Among the writers in our language who have of late contributed most successfully to the study of the borderland of philosophy and mathematics Bertrand Russell is perhaps the best known. In France M. Couturat has taken a prominent position, with the late lamented Poincaré writing with equal vigor in both regions. In Italy the writings of Peano, Pieri, and Veronese are well known, and other countries have contributed their quota to the study. It is, therefore, a helpful work that Dr. Mannoury has undertaken, to compile the views of various leading contributors to the study, while at the same time setting forth his own.

The work is divided into two parts, the first having to do with the foundations of arithmetic considered in its broadest sense, and the second with those of geometry. Under the former are considered in order the concepts of unity and multitude; of number, finiteness and infinity; of the distinctive fundamental principles of arithmetic; of the extension of the number concept and the principle of permanence; and of the irrational. As is often the case with continental writers the principle of permanence is attributed to Hankel, whereas Peacock introduced it in his Algebra nine years before Hankel