spicuous by its absence from the original, namely, the Klein-Poincaré representation of a non-euclidean plane on a euclidean plane by means of a system of circles orthogonal to a given circle.

In another new appendix (the fourth), the author shows very neatly how to construct projective geometry on the basis of Lobachefskian metrical geometry by adjoining ideal points, lines and planes. This meaning of the word "ideal" is sanctioned by common usage. In the fifth appendix, however, the term "ideal line" is used in a totally different sense, namely, for a circle which images or represents a straight line. This "double entendre" seems perhaps a trifle unfortunate. The translator has produced a very readable and satisfactory English version of the best historical introduction we have to the elements of non-euclidean geometry.

ARTHUR RANUM.

Dr. George Bruce Halsted—Géométrie Rationelle, Traité élémentaire de la Science de l'Espace—Traduction Française par PAUL BARBARIN, avec une preface de C. A. LAISANT. Paris, Gauthier-Villars, 1911. iv + 296 pp.

FROM the time of Farrar and Bowditch a number of French mathematical works have been translated into English, but although several American mathematicians have had their works translated into German, to Dr. Halsted belongs the honor of being the first to be translated into French. Novelties in geometry appeal to the French—witness their creations in connection with the geometry of the triangle, nomography, geometrography, anallagmatic curves and surfaces, and how Méray's somewhat radical work is coming to its own. As could, then, be almost predicted, when the first edition of Professor Halsted's book appeared in 1904 under the title "Rational Geometry, a Text-Book for the Science of Space based on Hilbert's Foundations," it was sympathetically received in France. Barbarin, already well known by his writings on non-euclidean geometry, wrote among other notices (of the first and second editions of Dr. Halsted's book) a ten-page review for Darboux's *Bulletin*.*

In Germany the work was not received so whole-heartedly and Dehn's somewhat vigorously expressed criticisms† (di-

* Sér. 2, vol. 31 (1907), p. 309–319.

† *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Nov., 1904, vol. 13, p. 592–596.

rected chiefly against discussion involving continuity and the style of treatment for a professedly elementary text) seem to have been recognized as just by the author. For, in the "thoroughly revised" and slightly abridged second edition (in which "Based on Hilbert's Foundations" is erased from the title page) changes are made at most points to which Dehn took exception, even in a case (mensuration of a circle) where Professor S. C. Davisson rather took issue* with Dehn.

A detailed account of the content and ideals of the later editions of Professor Halsted's strikingly original book would be merely a repetition of Professor Davisson's review which has already appeared in the BULLETIN.* I shall therefore confine myself to remarks on certain changes introduced into this latest edition and to comment on topics which do not seem to have been referred to in other reviews.†

The French edition is not, strictly speaking, a translation of either of those in English, but is rather a third edition based upon the second English edition. The titles are the same. The heading on page iv, "Preface de l'édition anglaise" is misleading, as the prefaces of both English editions have been combined and translated as that of an original work. The total of numbered paragraphs is the same as in the second English edition,‡ but several are new and revision has taken place throughout. There are 694 exercises instead of 700 and this change was made by leaving out some (e. g., 206, 692) and introducing others (e. g., 674–6). The French edition has only 184 figures while the second had 238. Although the table of contents is given in slightly expanded form, the index has been, unfortunately, left out. To the improvement of the work, Professor Halsted's "Table of Symbols" has been omitted and in the text and problems of the French edition only the ordinary abbreviations and symbols appear. While the English editions had as foot-note to Appendix I: "These proofs are due to my pupil, R. L. Moore, to whom I have been exceptionally indebted throughout the making of the book," the French edition merely gives, "Demonstration due à R. L.

* In the course of a review of the first edition of Professor Halsted's book in the BULLETIN, March, 1905, vol. 11, p. 330–336.

† Hathaway, *Science*, vol. 21 (1905), p. 183; *L'Enseignement Mathématique*, vol. 7 (1905), pp. 160 ff.; *Mathematical Gazette*, vol. 3 (1905), pp. 180–182. Vailati, *Boll. Bibliog. Storia Sc. matem.* (Torino), vol. 8 (1905), pp. 74–77. Eiesland, *Amer. Math. Monthly*, vol. 19 (1912), pp. 91–94. Baker, *Proc. Roy. Soc. Canada*, vol. 12 (1905), pp. 111–126.

‡ That is, counting paragraph 389, which was left out of this edition.

Moore, élève de G. B. Halsted." Barbarin was hard pressed to find single-word equivalents for some of Professor Halsted's terms and he omits such words as "symtra" and "tanchord." In other cases he introduces additional terms; for example, beside the word centroid (first adopted as synonymous with center of gravity by T. S. Davies*) he also gives "barycentre."

Professor Halsted associates names of mathematicians with theorems in perhaps a score of instances; for example: Archimedes (pages 70, 218), Pascal (page 73), Heron (page 110), Pappus (page 157), Euler (page 183), Lexell (page 255), Bordage (page 261), Joachimstal (*sic*, page 262). In practically no case (outside of the historical note on π) is there any indication when the mathematician lived, and in the whole book there is only a single exact reference to another book, namely, to Heiberg's text for Archimedes's axiom. Unless some such information is given in connection with all of the names I see no use in introducing them into a scientific work. And besides, which Heron is meant? Who unacquainted with less prominent French mathematical periodicals ever before heard of Bordage?

But if names are to be introduced, great care should be exercised. On page 278 Professor Halsted gives the example: "If from any point P on the circumcircle of a triangle ABC, PX, PY, PZ be drawn perpendicular to the sides, the points X, Y, Z are collinear on the Simson line of the triangle with respect to P." In the new (1911) edition of the Century Dictionary Professor Halsted gives this same definition. Now, for years it has been known† that this theorem is not to be found in either the published or unpublished work of Simson. On the other hand as William Wallace did enunciate the theorem the line should be known as Wallace's Line,‡ if special denomination be required.

Lexell's theorem is stated: The vertices of spherical triangles of the same angle-sum on the same base are on a circle copolar with the straightest bisecting their sides. I cannot find this in any of Lexell's papers. But the following theorem

* In 1843, *Mathematician*, vol. 1, p. 58. It had been used by Dr. Hey in 1814 to designate another point.

† *Proc. Edinburgh Math. Society*, vol. 9 (1891), p. 83. *Cf.* also Cantor, Vorlesungen über Geschichte der Mathematik, vol. 3 (1901), page 542.

‡ First remarked in 1885 by Dr. Thomas Muir (*Proc. Edinburgh Math. Soc.*, vol. 3, page 104). See my "Historical Note" in the same *Proceedings*, vol. 28 (1910), p. 64.

first discovered by Lexell was published in 1784:* The vertices V of spherical triangles ABV with the same angle-sum on the same base AB are on a certain small circle. Lexell gives† the following construction for this small circle: Through C the middle point of AB draw the great circle CZM, normal to the great circle AB and meeting it again in M; let the great circles AV, BV meet the great circle ABM in O and N respectively. Draw the great circle AXO making with ABO an angle equal to one half of the excess of the given angle-sum over 180°. Through O draw a great circle perpendicular to AXO to meet CZM in P; then with P as pole and distance PO describe a small circle and this is the required locus. In the course of his proof Lexell shows that $PO = PN = PV$. Hence the small circle NOV *is fixed by the points diametrically opposite to the ends of the base.* Steiner therefore incorrectly claims priority‡ in the discovery of this result.

Again, since 1860,§ it is clear that we should no longer refer to the theorem concerning convex polyhedra as the theorem of Euler, even though he enunciated and proved it as early as 1758.|| For, more than 75 years earlier the same theorem was formulated by Descartes. As Euler was merely a rediscoverer, the theorem should be known as the theorem of Descartes for polyhedra.

On the other hand in connection with the elegant but little known prismoidal two-term formula

$$V = \frac{h}{4}(B + 3S)\P$$

which "domine toute la mesure des volumes," the name of the Swiss mathematician Hermann Kinkelin who published the formula in 1862** might well be introduced. It was apparently rediscovered and discussed at least twice*** before it appeared

* *Acta academiæ scientiarum imperialis petropolitanæ pro anno* MDCCLXXXI, Pars prior . . . Petropoli, MDCCLXXXIV. "Solutio problematis geometrici ex doctrina sphaericorum," p. 112–126.

† *L. c.*, p. 123.

‡ *Crelle's Journal*, 1827, vol. 2, p. 45.

§ Oeuvres inédites de Descartes, éd. par Foucher de Careil, vol. 2, p. 214 ff.

|| *Novi Comment. Acad. Sc. petrop.*, vol. 4 (1752–1753), 1758, p. 109–160.

¶ Where h is the height, B the area of either end and S the area of the section two thirds of the distance from that end to the other.

** "Zur Theorie des Prismoides," *Archiv der Mathematik und Physik* (Grunert), vol. 39, p. 185.

*** J.K. Becker, "Einfachste Formel für das Volumen des Prismatoids," in Professor Halsted's Elementary Treatise on Mensuration, 1881.* Judging by the preface to the fourth edition of this work and by its denomination as "*New* prismoidal formula" it would appear that the author believed the result to originate with himself.

The historical note on π (pages 151–2) needs to be revised. Ludolf van Ceulen indicated the equivalent of the number π to 35 decimal places not 30.† Vega gave only 136 decimal places correctly, not 140.‡

Except for the lack of an index the French is a great improvement on the English edition. Apart from the figures (e. g., 58, 63, 91, 108, 113 are by no means up to the usual standard set by Gauthier-Villars) the pages are exceedingly attractive and it is to be hoped that a third English edition introducing still further improvements may not be long delayed. The work is full of interest and deals with a discussion of fundamentals in geometry, in an attractive style; and there can be little doubt that the number of universities using Professor Halsted's text in connection with courses on the Foundations of Geometry, will steadily increase.

The Japanese edition of the Rational Geometry which Sommerville lists§ as published in 1911 has not yet (in May, 1912) appeared.

R. C. ARCHIBALD.

Elementary Analysis. By P. F. SMITH and W. A. GRANVILLE. Boston, Ginn and Company, 1910. x + 223 pp.

THE number of textbooks in analytic geometry and calculus is rapidly increasing. But nearly all are intended for the use of students in engineering or for students who intend to specialize in pure or applied mathematics. In view, however, of the recent remarkable development of the natural sciences along mathematical lines, a brief course in analytic geometry and calculus is desirable for the general student who takes one year of mathematics as an elective beyond

Zeitschrift für Mathematik und Physik, vol. 23 (1878), p. 413 (dated Mai, 1877).—T. Sinram, *Archiv der Mathematik und Physik* (Grunert), vol. 63, p. 443 (November, 1878).

* Page 130.

† Bierens de Haan, *Messenger of Math.*, vol. 3 (1874), p. 25; copy of inscription on van Ceulen's tombstone.

‡ G. Vega, Thesaurus Logarithmorum, Leipzig, 1794, p. 633, and W. Shanks, Contributions to Mathematics, London, 1853, p. 86.

§ Bibliography of Non-Enclidean Geometry, 1911.