

proper place in the main body of the text. Besides, the appendix contains two tables to serve as an aid in properly classifying and integrating differential equations.

The book is well written and is well adapted as a general introduction to Lie's theory. As already indicated, the too systematic treatment of differential equations from this point of view appears to the reviewer to be a defect. It is one thing to recognize that Lie's theory is capable of bringing some order into the haphazard methods of the elementary theory of differential equations; but it is quite another to use Lie's theory as a strait jacket which every differential equation must be made to fit.

And now for a few points of detail. The author's definition of a one-parameter group is rather clumsy, as a result of the unfortunately so common desire to put it all into one sentence. The two parts of remark 1 on page 7 are not obviously connected. The true reason for the validity of remark 1 is to be found in the latter part of remark 2. Although the author speaks everywhere of one-parameter groups, he really means such groups as are generated by an infinitesimal transformation. At least, a number of his theorems are not true for mixed groups. On page 176 in deriving the condition for a "union of lineal elements" he neglects the case where the union consists of the elements of a fixed point. The printing is well done. The reviewer has noticed only a few misprints. On page 45 last line read  $-A\eta$  instead of  $+A\eta$ . Page 177 in Ex. 5 read  $-\cot t$  instead of  $-\cos t$ . Page 188 just above equation (130') read characteristic instead of charactertisic.

A rather confusing feature is the use of two different scarcely distinguishable fonts for the same letters, as on page 24. There is no necessity for this. It seems a pity that the publishers have not seen fit to give the book a wider margin. The binding also is none too good.

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*Abriss der Algebra der Logik.* Bearbeitet im Auftrag der Deutschen Mathematiker-Vereinigung von Dr. EUGEN MÜLLER, Professor an der Oberrealschule zu Konstanz. In drei Teilen. Erster und Zweiter Teil. Leipzig und Berlin, B. G. Teubner, 1909-1910, 159 pp: 8 marks.

ALMOST from the days of Boole an ever-increasing need has been felt for an elementary introduction to the growing subject

of the algebra of logic. There have been numerous extended and more or less exhaustive treatises on the subject, notably that of Dr. Ernst Schröder, but few of an elementary character.\* The present work has the name of Dr. Schröder at the top of title page and was inspired by the references in several places in Schröder's Vorlesung to a proposed "Abriss." The first part, which serves as an introduction, was partly prepared from an outline and manuscript fragments left by Schröder. The first twenty pages are given up to the elementary introduction to the ideas, terms, and symbols used. The standpoint used here is that of the algebra of propositions or implications. This part of the book is rather diffuse and the entire work contains much that is obvious, and could be condensed with advantage. This is probably intended to appeal to the non-mathematical or philosophical reader. The seven axioms on which the subject is based are given on pages twenty and twenty-one. This part is interpreted from the standpoint of the algebra of inclusion or classes or of incident regions, and the two phases are treated from this point on in parallel.

The axioms are as follows:

- I.  $a \notin a$ , Identitäts- oder Tautologieaxiom.
- II.  $(a \notin b)(b \notin c) \notin (a \notin c)$ , Subsumtionsschluss.
- III.  $(a \notin b)(b \notin a) = (a = b)$ , Gleichheitsdefinition.
- IV $\times$ .  $0 \notin a$ , Nullpostulat. IV $+$ .  $a \notin 1$ , Einspostulat.
- V.  $1 \notin 0$  Existenzpostulat.
- VI $\times$ .  $(x \notin a)(x \notin b) = (x \notin ab)$ , Produktdefinition.
- VI $+$ .  $(a \notin y)(b \notin y) = (a + b \notin y)$ , Summendefinition.
- VII $\times +$ .  $(a + z)(\bar{a} + z) = z = az + \bar{a}z$ , Negations- oder Distributionsprinzip.

These axioms are explained and some alternate axioms are given in the next five pages. The rest of part one (to page 50) is taken up with the deduction of theorems which are the immediate consequences of these axioms. Most of the relations in part one are expressed in terms of the asymmetrical sign of implication or inclusion under the general term of "subsumption" ( $\notin$ ), and the sign of equality is defined by Axiom III in terms of the sign of subsumption. There is a logical fallacy involved here in using the equality sign in its

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\* Except the work by Couturat I know of no other of a general character.

own definition.\* A better definition would be composed of the two parts†

$$(a = b) \notin (a \notin b)(b \notin a), \quad \text{and} \quad (a \notin b)(b \notin a) \notin (a = b).$$

The equality sign is used however before Axiom III is given. In part two, *Aussagentheorie, Funktionen, Gleichen und Ungleichen*, the author derives and discusses at length the general formulas for the symbolic sum and product, and the methods of substitutions and reduction of symbolical functions with the methods of elimination and solution of the general symbolical equation. This part is quite extensive and there is much that is obvious.

One good feature of the work, at least from the standpoint of the mathematician, is the numerical or algebraical examples, illustrative of the general theory, used throughout the book. The *Abriss* on the whole is a very creditable piece of work and cannot but help arouse interest in the algebra of logic. No serious errors occur, but there are some obvious misprints of the type found in line three, page thirteen, where  $a$  in the last parenthesis should be  $\alpha$ .

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*Cours d'Astronomie. Première partie: Astronomie théorique.*  
Par H. ANDOYER. Deuxième édition entièrement refondue.  
Paris, Librairie scientifique A. Hermann. 383 pp.

THE first edition of this work in hectographed form has been reviewed in volume 13, number 10, of the *BULLETIN*.‡ The second edition is much enlarged, almost doubled in size. The number of chapters is increased from fifteen to nineteen. The work in its present form is subdivided into four large sections, or books, of which the first is of a more analytical character. The transformation of coordinates with common origin—spherical trigonometry—is given with considerable completeness; to it is added the transformation for different origins and parallel systems of axes—the analytical basis for parallax and aberration. The second book defines the adopted systems of coordinates in astronomy, the concepts of sidereal and solar

\* A similar fallacy is evident in axiom VI $\times$ . It would be difficult to avoid this in a simple way.

† A combination of his III', III'' and III'''.

‡ See also the review of the second part, by Professor Longley, *BULLETIN*, vol. 15, pp. 467-468.