

## NON-EUCLIDEAN GEOMETRY.

*Bibliography of Non-Euclidean Geometry including the Theory of Parallels, the Foundations of Geometry, and Space of  $n$  Dimensions.* By DUNCAN M. Y. SOMMERVILLE. London, Harrison and Sons, 1911. xii+404 pages.

ALMOST simultaneously with its quincentenary celebration, the University of St. Andrews has published this valuable compilation, the result of nine years of research on the part of one of its lecturers in mathematics. The title inevitably suggests similar lists by Halsted, Stäckel and Engel, Schlegel, and the lamented Bonola. To the work of all these bibliographic forebears Dr. Sommerville gives scrupulously exact reference. The bibliography of Halsted, "a model of its kind," includes nearly 200 titles of works relating to non-euclidean geometry and space of  $n$  dimensions, from about 1830 to 1879. Short notes appended to the chief works "form a valuable feature." Stäckel and Engel's bibliography on the theory of parallels is a chronological list of nearly 300 titles from 1482 to 1837 and, as Dr. Sommerville remarks, is almost complete. Schlegel's list of works on  $n$  dimensions accompanies a report on the subject, which supplies the place of Halsted's notes. It contains about 400 titles arranged alphabetically under the authors. The bibliography of Bonola is the most extensive which has recently appeared. It contains over 900 titles chronologically arranged from 1839 to 1902 with an index of authors. There is a classification under the headings Elementary geometry, Metrical and differential geometry, Group theory, Projective geometry, Finite distances, Vectorial methods, Mechanics and physics, General expositions, Philosophy and history, but there is no subject index.

Dr. Sommerville's idea at first was to prepare a continuation of Halsted's work but the growth of the subject rendered such diffuse treatment impossible and he was led to "produce as far as possible a complete repository of the titles of all works, from the earliest times up to the present, which deal with the extended conception of space, and to form a guide to the literature in an easily accessible form. It includes the theory of parallels, non-euclidean geometry, the foundations of geom-

etry and space of  $n$  dimensions. Works on the foundations of mathematics or arithmetic are excluded except in so far as they have an explicit reference to geometry."

The titles of which Dr. Sommerville takes account are over 4,000 in number. They may be roughly classified as follows: theory of parallels 700, non-euclidean geometry and the foundations of geometry 1,600,  $n$  dimensions 1,800. The approximate analysis of the nationalities of the authors (excluding mere reviewers and translators) gives: German, 460; French, 230; Italian, 220; British, 160; American, 90; Russian and Polish, 80; Dutch, 35; Hungarian, Bohemian, Croatian, etc., 35; Scandinavian, 35; no attempt is made to distinguish between German and Austrian or between Frenchman, Belgian, and Swiss. The total number of titles in the German language is 1,159; in French 884, Italian 848, English 723, while other languages represented by less than 100 titles are: Russian 99, Latin 80; Dutch 79, Magyar 37, Spanish and Portuguese 27, Danish and Norwegian 27, Polish 22, Swedish 11, Croatian 9, Greek 9, Czech 6, Arabic 4, Esperanto 4, Ruthenian 2, Roumanian 1, Sanskrit 1. The fact that no less than 1,470 titles are assigned to the decade 1901-1910 is an interesting indication of the trend of mathematical research at that time.

Riccardi's *Bibliografia Euclidea* was the model followed by Dr. Sommerville in the division of his work into three parts: I (pages 1-260) Chronological Catalogue, which covers the period from the time of Aristotle and Plato, who in the fourth century B.C. discussed the parallel postulate and the definitions of point and line, down to June, 1911, although no general attempt was made to obtain titles of later date than 1910; II (pages 251-316) Subject Catalogue in which the classification of the 1908 edition of the *Index du répertoire bibliographique* is used in amplified form; III (pages 317-398) Author's Index which, as far as possible, gives full names together with year of birth and, in the case of deceased authors, of death. Under each author's name is an abbreviated title and date of each of his writings, the details concerning which are to be found in I.\* In the chronological catalogue the titles in each year are arranged alphabetically according to the authors. The various

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\* The seven persons who have the greatest number of titles (including biographical sketches and reviews) to their credit in Dr. Sommerville's list are, in order: Halsted (77), Mansion, Schoute, Segre, Poincaré, Klein, Bianchi.

editions, translations and reviews \* of a work as well as its classification (II) and price, when a separate work, are here found. References to bibliographical sources have only been given in cases of discrepancies among the authorities. A very few titles are marked with an asterisk (\*) as having been personally verified by the author. This fact naturally inspires the fear that the subject classification will require considerable emendation to make it an accurate epitome of the work done in the various fields. On pages 399-404 are given "additions" and "corrections."

It may be asked, to what extent has the author succeeded in making a complete work within the prescribed limits? On looking through the list of his sources of information we find beside the bibliographies mentioned above, the *Jahrbuch; Revue Semestrielle*; International Catalogue (A); Royal Society Catalogue, Authors, volumes 1-12, Subject, volume 1; † Poggendorff; and Wölffing's *Mathematischer Bücherschatz*. It is with some surprise that we learn that no use was made of the Royal Society Catalogue, Subject Index, volume 2, "Mechanics," which would have supplied at least four additional titles of value, or of the International Catalogue, (B) Mechanics, (C) Physics, or of the 1,900 slips of the *Répertoire bibliographique des Sciences mathématiques*. Nor do we anywhere find references to Felix Müller's excellent *Führer durch die mathematische Literatur* with its well arranged geometry bibliographies which cover many pages. We are therefore prepared for numerous omissions (the more, when we recall the well known imperfections of most of the above mentioned works) and it is indeed probable that they number many hundreds, even though Dr. Sommerville has already digested a great mass of material. A very brief examination of his book suggested a score of additions: ‡ D. Lardner, *Elements of Euclid*, third edition, London, 1832; sixth edition, 1838; eighth, 1843; ninth, 1846; tenth, 1849; W. C. Hume, *A treatise on the theory of parallel lines*, Dublin, 1853, pages viii + 40 and 2 plates; E. Padova [extension to  $n$ -space of two theorems by Neumann on potential], *Giornale di Matematiche*, Napoli, volume 8 (1870), pages 296-301; J. König [*Traité ana-*

\* It is to be regretted that the names in connection with signed reviews are rarely given.

† Dr. Sommerville has, apparently, overlooked the Appendix of this volume.

‡ It is to be understood, of course, that not all the following titles would be noticed except in a work aiming to be as complete as the one under review.

lytique des hypothèses de la géométrie], *Értekezések a Mathematikai Osztály Koréből. Kiadja a Magyar Tudományos Akadémia*, Budapest, volume 6 (1872), pages 94–98; A. Tonelli [Potential function in  $n$ -space], *Göttinger Nachrichten*, 1875, pages 521–552; É. Lemmi, “Sur les cas d’exception au théorème des forces vives . . .,” *Journal de Mathématiques pures et appliquées*, Paris, volume 2 (1876), pages 233–239; Réthy [Trigonometry in non-euclidean space], *A Magyar Tudományos Akadémia Ertesítője*, Budapest, volume 25 (1876); F. W. E. A. Kellner [Considerations on the theory of parallels as basis of geometry], Amsterdam, 1879, 81 pages; O. Simony, “Ueber eine Reihe neuer mathematischer Erfahrungssätze,” *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der Kaiserlichen Akademie der Wissenschaften*, Wien, volume 87 (1883), pages 556–587; volume 88 (1884), pages 939–974; F. M. Suvorof [Sur la formule générale pour la distance de deux éléments dans un système projectif à une dimension], *Proceedings of the Physico-Mathematical Section of the Naturalists’ Society of the Imperial University of Kazan*,\* volume I (1883) pages 9–11; J. C. Medeiros [Complément à la théorie des parallèles d’Euclide], *O Instituto, jornal scientifico e litteraris*, Coimbra, volume 30 (1883), pages 541–553; L. Kronecker [Clausius’s coordinates in  $n$ -space], *Berliner Sitzungsberichte*, 1891, pages 881–890; Klein, “Vorlesungen über projektive Geometrie” (mimeographed), Cambridge, Mass., 1893;† A. Vasiliev, N. I. Lobachevsky, *El Progreso matematico*, volume 3 (1893), pages 137–139; J. N. Lyle, various papers in the *American Mathematical Monthly*, volumes 1–3 (1894–1896); in volume 3 (1896) of the *Monthly* there was also a paper by J. H. Drummond, in volume 11 (1904) one by E. B. Wilson, in volume 16 (1909) one by Wilczynski. No reference is given to Klein’s “Elementarmathematik vom höheren Standpunkte aus,” Teil II: Geometrie, Leipzig, 1909, or to an 8 page (large quarto size) sketch (in French) of Lobachevsky’s life and works published, according to a pencilled note on my copy, at Kazan in 1886. The greater part (pages 581–593‡) of Note II (“Sur la géométrie non-euclidienne”) in the seventh edition of Rouché and Comberousse’s *Géométrie*, volume 2, Paris, 1900,

\* Translation of Russian title.

† The edition of about 25 copies was published by some graduate students of Harvard University.

‡ E. Lebon in the Poincaré volume of the *Savants du Jour* series makes a slip in connection with this entry.

should have been credited to Poincaré. "Rouché and Comberousse" is incorrectly listed under the date 1891, of its sixth edition; in the fifth edition of 1883 the notes treat of the same subjects as in the later editions. Miss Scott's articles in volume 1 (1900) of the *Mathematical Gazette* should surely be mentioned in connection with von Staudt's *Geometrie der Lage*, and if two English editions of Newcomb's *Popular Astronomy* are to be listed, reference should also be given to the 8 American, the 4 German, the Russian, and the Norwegian editions. The entry after "[Crelle, A. L.]" under 1835 should be put in the list for 1834. To the title under Eisenmenger's name on page 52 may be added: pp. 14+1 pl.

Incomplete as the work before us therefore appears, the mathematician must ever be grateful to Dr. Sommerville for this new, carefully prepared and admirably arranged aid to orientation in a vast field of mathematical science.

The volume is well printed by His Majesty's Stationers and the general get up (including the paper binding) is the same as for the *International Catalogue*.

R. C. ARCHIBALD.

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## NOTES.

THE thirtieth regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will be held at the University of Chicago on April 5-6.

THE opening (January) number of volume 13 of the *Transactions of the American Mathematical Society* contains the following papers: "Ueber eine idealtheoretische Funktion," by E. LANDAU; "Theorems of oscillation for two linear differential equations of the second order with two parameters," by R. G. D. RICHARDSON; "The absolute minimum of a definite integral in a special field," by E. J. MILES; "An existence theorem for a problem of the calculus of variations in space," by E. G. BILL; "Linear algebras," by L. E. DICKSON; "A note concerning Veblen's axioms for geometry," by R. L. MOORE; "Natural families of curves in a general curved space of  $n$  dimensions," by JOSEPH LIPKE; "A class of periodic orbits of superior planets," by F. R. MOULTON; "Harmonic functions and Green's integral," by O. D. KELLOGG.