THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and fifty-fifth regular meeting of the Society was held in New York City on Saturday, October 28, extending through the usual morning and afternoon sessions. The attendance included the following thirty-five members:

Professor W. J. Berry, Professor G. D. Birkhoff, Professor G. A. Bliss, Professor E. W. Brown, Professor B. H. Camp, Professor F. N. Cole, Dr. G. M. Conwell, Dr. H. B. Curtis, Dr. L. L. Dines, Professor L. P. Eisenhart, Professor H. B. Fine, Professor T. S. Fiske, Professor W. B. Fite, Professor W. A. Garrison, Professor C. C. Grove, Professor E. V. Huntington, Mr. S. A. Joffe, Professor Edward Kasner, Professor C. J. Keyser, Dr. N. J. Lennes, Professor James Maclay, Dr. Emilie N. Martin, Dr. H. H. Mitchell, Dr. Anna J. Pell, Dr. H. W. Reddick, Professor R. G. D. Richardson, Miss S. F. Richardson, Professor Paul Saurel, Mr. L. P. Siceloff, Professor P. F. Smith, Professor H. D. Thompson, Dr. M. O. Tripp, Professor Oswald Veblen, Mr. H. E. Webb, Miss E. C. Williams.

President Fine occupied the chair at the two sessions. The Council announced the election of the following persons to membership in the Society: Professor T. B. Ashcraft, Colby College; Professor Clara L. Bacon, Goucher College; Professor J. M. Davis, State University of Kentucky; Professor W. C. Eells, Whitworth College; Dr. J. L. Jones, Yale University; Professor F. C. Kent, University of Oklahoma; Professor L. C. Plant, University of Montana; Mr. R. E. Powers, Denver, Colo.; Mr. T. M. Simpson, University of Wisconsin; Professor Evan Thomas, University of Vermont; Professor H. C. Wolff, University of Wisconsin; Mr. W. A. Zehring, Purdue University. Nine applications for membership in the Society were received.

A list of nominations for officers and other members of the Council, to be placed on the official ballot for the annual meeting, was adopted. Committees were appointed to make arrangements for the summer meeting of the Society at the University of Pennsylvania, in 1912, and to audit the Treasurer's accounts. The invitation of the University of Wisconsin to hold the summer meeting and colloquium at that University in 1913 was accepted.

A committee consisting of President Fine as chairman and Professors E. W. Brown, C. J. Keyser, E. H. Moore, W. F. Osgood, and E. B. Van Vleck was appointed to consider and report to the Council a plan for placing the business of the Society on a permanent basis. It was decided to change the form of the Annual Register by omitting in the personal entries all mention of membership in other organizations.

The following papers were read at this meeting:

(1) Mr. A. R. SCHWEITZER: "On a functional equation."

(2) Professor E. V. HUNTINGTON: "A new approach to the theory of relativity."

(3) Mr. L. P. SICELOFF: "Simple groups from order 2,001 to order 3,640."

(4) Dr. H. H. MITCHELL: "Determination of the quaternary linear groups by geometrical methods."

(5) Professor G. A. BLISS: "A new proof of the existence theorem for implicit functions."

(6) Mr. R. E. POWERS: "The tenth perfect number."

(7) Professor E. W. BROWN: "On the summation of a certain triply infinite series."

(8) Dr. L. L. DINES: "On the highest common factor of a system of polynomials."

(9) Professor R. D. CARMICHAEL: "Fundamental properties of a reduced residue system mod m."

(10) Professor R. D. CARMICHAEL: "A generalization of Cauchy's functional equation."

(11) Professor R. D. CARMICHAEL: "On composite numbers P which satisfy the Fermat congruence $a^{P-1} \equiv 1 \bmod P$."

(12) Professor EDWARD KASNER: "Differential invariants of infinite order."

(13) Professor B. H. CAMP: "Series of Laplace's functions."

(14) Dr. N. J. LENNES: "A new proof that a Jordan curve separates a plane."

In the absence of the authors the papers of Mr. Schweitzer, Mr. Powers, and Professor Carmichael were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Mr. Schweitzer proves the following theorem:

Let $u = \phi(x, z)$, $v = \phi(x, y)$ and $w = \phi(y, z)$; then if $\phi(u, v) = w$, there exists a function χ such that $\chi(v) = \chi(x) - \chi(y)$. A similar result is valid in case of the relation $\phi(u, w) = v$.

The method of proof is analogous to that of Abel * and Stäckel † who show (in effect) that if $\phi(y, u) = \phi(z, v) = \phi(x, w)$, then there exists a function χ such that $\chi(v) = \chi(x) + \chi(y)$.

2. The theory of relativity as developed by Einstein ‡ is usually supposed to involve a radical modification, not only of our conception of the ether, but also of our fundamental notions concerning space and time; and the discussion of the so-called "paradoxes of relativity" has often led beyond the safe ground of mathematical deduction into the realm of metaphysical speculation.

The purpose of Professor Huntington's article is to show that the famous "transformation equations," which stand at the center of the theory, can be easily deduced from simple conventions in regard to the setting of clocks and the laying out of coordinate systems, without any conflict with our ordinary conception of the ether, or with our ordinary notions of space and time, thereby freeing the theory from the least appearance of paradox.

The method pursued is in some sense a return to the point of view of Lorentz, § who retained the concept of the ether as his starting point, and never abandoned our ordinary notions of time and space; and the transformation equations here obtained resemble the equations of Lorentz in being slightly more general than those of Einstein. Lorentz's method of deducing these equations, however, involves a large and difficult part of the modern electromagnetic theory, while the method here adopted depends only on the most elementary considerations.

The present article is purely expository, and does not attempt any critical or historical discussion. ||

3. In this paper Mr. Siceloff continues the search for new simple groups of low order. He finds that the only orders of simple groups between 2,001 and 3,640 are those already well-

* *Crelle's Journal*, vol. 1 (1826), p. 11.

† *Zeitschrift für Mathematik und Physik*, vol. 42 (1897), p. 323.

‡ A. Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik*, ser. 4, vol. 17 (1905), pp. 891–921.

§ H. A. Lorentz, Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern, Leiden, 1895. Neudruck, Leipzig, 1906. Also, The Theory of Electrons (Columbia University lectures, 1906), Leipzig, 1909.

|| For an extensive bibliography of the theory of relativity, the reader is referred to an article by J. Laub, in the *Jahrbuch der Radioaktivität und Elektronik*, vol. 7 (1910), December, p. 405.

known, viz., 2,520 and 3,420. Whether there is a second simple group of order 2,520 has not been determined. The results are based chiefly on the study of the Sylow subgroups.

4. A complete enumeration of all primitive collineation groups in ordinary space of three dimensions has been given by Blichfeldt (*Mathematische Annalen*, 1905). This was done primarily by algebraic methods and involved the use of theorems concerning the irreducibility of equations containing roots of unity. It is the purpose of Dr. Mitchell's paper to determine these groups by geometrical methods and in such a way as to determine also the quaternary groups where the coefficients of the transformations are marks of the $GF(p^n)$, but the order of the group is not divisible by the prime modulus p. Properties of the linear complex are found to be of much service in the solution of the problem.

5. The paper of Professor Bliss has to do with the solution of a system of equations of the form

$$f_i(x_1, x_2, \cdots, x_m; y_1, y_2, \cdots, y_n) = 0 \quad (i = 1, 2, \cdots, n).$$

The theorem stating the existence of a set of solutions of the form

$$y_i = y_i(x_1, x_2, \cdots, x_m) \quad (i = 1, 2, \cdots, n)$$

for these equations is an old one, but in the well-known proofs of it a single equation is usually considered first, and then the proof is made for a system of equations by applying the principle of mathematical induction. In the present paper a proof is given which seems relatively simple, and which applies with equal convenience to either one or more equations.

6. The composition of Mersenne's numbers $2^p - 1$, p a prime, is known for all values of p not exceeding 100 with the exception of $p = 89$. E. Lucas, in volume 1 (1878) of the *American Journal of Mathematics*, proved the following theorem: "If $N = 2^{4q+1} - 1$, and if we form the series of residues (modulo N) 4, 14, 194, $\cdots$, each of which is equal to the square of the preceding, diminished by two units, the number N is prime if the first residue 0 lies between the $2q$th and the $(4q + 1)$th term." Mr. Powers shows that for $4q + 1 = 89$ the 88th term of the foregoing series is 0 (first 0); hence $2^{89} - 1$ is a prime number, and $2^{88}(2^{89} - 1)$ is the tenth perfect number.

7. In the expansion of Hill's infinite determinant there occurs a term which is equivalent to

$$\Sigma_i \Sigma_k \Sigma_{k'} \{i\}\{i+1\}\{i+k\}\{i+k+1\}\{i+k+k'\}\{i+k+k'+1\},$$

where each of the six factors is obtained by giving j the proper value in $1/\{j\} = \alpha^2 - j^2$. Here α is unequal to any integer, i takes all integral values from $+\infty$ to $-\infty$, and k, k' all integral values from 2 to ∞. Hill gives the value with but slight indication of the method by which he obtained it. Professor Brown shows how it may be obtained without great labor, and is thus able to verify Hill's result.

8. In a paper before the Chicago meeting of the Society in April, Dr. Dines considered the conditions characterizing the existence of a common factor of first or higher degree of a system of n polynomials in one variable. It was found that, if a certain restriction be made upon the coefficients, then necessary and sufficient conditions can be stated in terms of the vanishing of $n - 1$ determinants.

In the present paper, a matrix is constructed whose elements are the coefficients of the given polynomials arranged according to a simple rule, and which for $n = 2$ reduces to the matrix of the well known Sylvester resultant of two polynomials. This matrix possesses the following properties:

(1) The vanishing of the matrix constitutes necessary and sufficient conditions for the existence of a common factor of first or higher degree. (A matrix of μ rows and ν columns, $\mu \leqq \nu$, is said to vanish when every determinant of order μ of the matrix vanishes.)

(2) The degree of the highest common factor of the system of polynomials can be stated in terms of the rank of the matrix.

(3) The coefficients of the highest common factor are determinants of the matrix, easily characterized.

(4) There are always $n - 1$ determinants of the matrix whose vanishing is equivalent to the vanishing of the matrix.

9. The most important example of an abelian group is furnished by the $\varphi(m)$ integers of a reduced residue system mod m, the operation of combination being multiplication and reduction mod m. In the present paper Professor Carmichael determines the invariants of such a group and gives a method for constructing all groups of this class having the same in-

variants as a given group of the class. In this way all groups of the class, simply isomorphic with a given one, are determined. Let us define $\lambda(m)$ as the greatest exponent to which any element of the reduced residue system mod m belongs. A method is given for finding every x such that $\lambda(x) = \lambda(m)$, and incidentally for finding every x such that $\varphi(x) = \varphi(m)$. The paper closes with a table giving all possible solutions of $\lambda(x) = a$ for every possible value of a up to $a = 24$. The solutions are separated into sets according to the type of group defined by the corresponding reduced residue system. In each case the invariants of the group are given.

10. The nature of the generalization of Cauchy's functional equations effected in Professor Carmichael's second paper is indicated by the following theorem: If α is any positive or negative quantity and u_z is any function of z capable of assuming at least once each value in the set

$$2^{-s}n\alpha \quad (n, s = 0, 1, 2, \cdots; \; n \leqq 2^s),$$

then the most general function $f(x)$ which is continuous on the interval $(0, \alpha)$ and satisfies the functional equation

$$f(x + u_z) = f(x) + f(u_z)$$

is $f(x) = ax$, where a is an arbitrary constant.

11. In his note on the Fermat congruence $a^{P-1} \equiv 1 \bmod P$ Professor Carmichael gives a method of finding composite numbers P which satisfy the congruence for every a which is prime to P.

12. When an analytic curve is referred to the tangent and normal at one of its points, its equation takes the form

$$y = a_2x^2 + a_3x^3 + \cdots.$$

The coefficients are differential invariants of the curve, the first being half the curvature. Every combination of the coefficients is an invariant, that is, intrinsically related to the curve. Professor Kasner considers combinations involving an infinitude of coefficients. An important example is the radius of convergence of series y. Another invariant is the quantity J defined as the first non-vanishing term in the sequence $a_2, a_3, \cdots$. This vanishes only when the curve is a straight line, and is applied to certain questions concerning geodesics and trajectories. The even and odd components of a curve at a given point, introduced by the author in an earlier paper, are examples of covariants of infinite order.

13. The most important theorem concerning the validity of the expansion of an arbitrary function in a series of Laplace's functions has been proved by Jordan in his Cours d'Analyse, second edition, volume 2, page 252. The conditions stated there are that the given function be continuous on the surface of the sphere within some small circle about the point at which the expansion is made, and that it have limited variation along every great circle through the point.

The object of Professor Camp's paper is to correct an error in Jordan's theorem, and to furnish new sufficient conditions for the validity of these expansions. To the conditions announced by Jordan should be added the requirements that the values of the variations be all less than some fixed number, and that these variations be uniform with respect to all great circles through the point. Furthermore, the conditions of continuity of the function and of uniformity of the variations may be replaced by the different condition that there exist a small circle about the point considered such that within it the given function is, along all great circles through the point, an indefinite integral of another function which has an absolutely convergent double Lebesgue integral in its domain of definition. Any or all of these conditions may fail on a null set of great circles through the point.

14. In a paper presented to the Society at the April meeting Dr. Lennes proved that a closed continuous surface separates a three-space in which it lies into two connected sets. The present paper is an adaptation of the methods used in that paper to the case of the curve in the plane.

F. N. Cole,
Secretary.