SHORTER NOTICES.

Diophantus of Alexandria. A Study in the History of Greek Algebra. By Sir THOMAS L. HEATH. Second Edition. Cambridge, University Press, 1910. vi + 387 pp.

THE edition of 1885 has been modified to such an extent that the present edition is in effect a new work. Most of the introductory chapters have been entirely rewritten. The account of the Arithmetica is fuller and a more literal translation of the original; the critical text by Tannery (Teubner, 1893, 1895) has been drawn upon to remove the former obscurity of certain passages.

The first edition paid little attention to the very important annotations by Fermat upon his copy of Bachet's edition (1621) of Diophantus. This deficiency has been remedied in the present work, which gives also much material selected from the correspondence of Fermat (Oeuvres, Paris, 1891–1896). Various investigations by Euler relating more or less closely to the problems discussed in the Arithmetica are given either as footnotes or in the final chapter, which devotes 52 pages to solutions by Euler. These extensive additions enable the reader to see in a concrete manner the influence which the work of Diophantus has had upon the development of the theory of numbers. Diophantus originated the theory of indeterminate equations; but in their solution he imposed the same limitations as in the case of determinate equations, requiring positive rational solutions. The restriction to integral solutions was made only in special cases, so that the present day designation of Diophantine equations is a misnomer.

It is stated on page 107 that "it is not probable that Diophantus was aware that prime numbers of the form $4n + 1$ and numbers arising from the multiplication of such numbers are the only classes of numbers which are always the sum of two squares." A factor 2^k may also be present.

On page 108 Heath states that we must credit Diophantus with the knowledge that no number of the form $8n + 7$ can be the sum of three squares. This is an illogical inference from the statement of Diophantus to the effect that no number of the form $3a + 1$, where $a = 8k + 2$, is the sum of three squares;

Diophantus makes no statement whatever about numbers of the forms $24n + 15$, $24n + 23$.

The limit to the numbers for which Bachet verified that each is the sum of at most four squares is stated to be 120 on page 110 and to be 325 on page 188.

On page 293, line 13 from bottom, Fermat is quoted as saying, "But both these squares can be shown to be smaller than the squares···," whereas the translation should be, "But the sum of these two squares can be shown to be smaller than that of the first two···" (Oeuvres, 3, page 272). The former reading would change the nature of the proof, and is moreover not in accord with the next sentences of the quotation. The same incorrect version occurs in the first footnote on page 295, in which Heath points out that Zeuthen changed (in another manner) the reasoning of Fermat.

While in the first edition the numbering of the problems in each book of the Arithmetica was in accord with the usage of other writers, this is unfortunately not the case with the present edition. Sections which give lemmas needed in subsequent sections are left unnumbered (and so in effect fall into the irrelevant earlier sections). While it is certainly desirable to have the heading which indicates this fact that the section is a lemma to a later section, the omission of a number for the section is a defect. But of course the chief objection is the lack of uniformity with established usage, which lessens the value of the work as a reference book. In histories of mathematics and in texts on the theory of numbers there occur references to the comments by Bachet or Fermat on Diophantus IV, 20, for example. Changes in the standard numbering are most unfortunate.

It was only after a careful examination that the reviewer was able to locate these few defects. It is obvious that the book is the outcome of a great amount of careful investigation on the part of an expert on the earlier and later history of mathematics. The result is an attractive and trustworthy account of the work of Diophantus and a comprehensive analysis of its historical setting and its influence upon the subsequent development of algebra and the theory of numbers.

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