particular one on the totality of primes comprised within given limits.

The memoir on subinvariants and perpetuants belongs to this period. At the end of the volume appear some short papers on the theory of partitions; but Sylvester's chief work in this field will appear in the fourth (final) volume of his collected papers.

L. E. DICKSON.

Niedere Zahlentheorie. Zweiter Teil. By DR. P. BACHMANN. B. G. Teubner's Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Band X, 2. Leipzig, 1910. x + 480 pp.

THE first volume of this work appeared in 1902 and was reviewed by the writer in the BULLETIN, volume 9 (1903), page 555. The present second volume has the subtitle Additive Zahlentheorie, a term suggested by Kronecker for the properties of numbers relating to their additive combinations.

The first chapter deals with arithmetical series of the nth order, polygonal and figurate numbers, the sum of the kth powers of the first n integers, the sum of the kth powers of n numbers in arithmetical progression, and at length with the theory of Bernoullian numbers. The second chapter deals with recurring series, in particular with those bearing the names Farey, Fibonacci, Fermat, Pell, and Dupré. Application is made to factorization of numbers of certain forms, perfect numbers, Fermat and Mersenne numbers. Here results are omitted that have been known for several years. For instance the number $2^{67} - 1$ is left in doubt although Cole has given two factors (BULLETIN, 1903, page 137). Chapters III–V relate to the theory of partitions. Chapter VI is entitled recursion formulæ and deals with various number theoretic functions. Chapters VII–VIII treat of the representation of a given number as a sum of like powers or as a sum of multiples of powers. Every positive integer can be expressed as the sum of four squares (but not always as the sum of three); as the sum of nine cubes (but not always as the sum of eight); as the sum of 37 fourth powers (though doubtless this limit is too high). Further theorems relate to the number of representations of an integer as the sum of four squares. The related investigations by Liouville are given at length. The final chapter deals with Fermat's equation $x^n + y^n = z^n$ from the standpoint of the elementary theory of numbers, giving Fermat's proof for $n = 4$, the Euler-Legendre proof for $n = 3$, and remarks on the Dirichlet-Legendre proof for $n = 5$, and the Lamé-Lebesgue proof for $n = 7$. Kummer's method by ideals is beyond the scope of the work; the comment on regular primes (page 461) is corrected at the end of the book. The formulas obtained independently by Abel and Legendre are established. The developments by Sophie Germain, E. Wendt, and L. E. Dickson are then cited. In one instance (page 475), the initials of the last name are given incorrectly; while, in a quotation from Sylvester on page 104, permeating is spelled wrong. However, the book is especially free of errata and the typography is excellent. In the present text Bachmann has fully maintained his reputation as to clearness, thoroughness, and exhaustiveness.

L. E. DICKSON.

Eléments de Calcul vectoriel avec de nombreuses Applications à la Géométrie, à la Mécanique et à la Physique mathématique. Par C. BURALI-FORTI et R. MARCOLONGO. Traduit de l'italien par S. LATTÈS. Paris, A. Hermann et Fils, 1910. vii + 229 pp.

So lengthy a review * was recently accorded to two new books on vector analysis by Burali-Forti and Marcolongo that nothing more than the mere mention of the French edition of the first of the two would be needed, were it not for the fact that in the French the authors have added a long and excellent appendix on Grassmann's geometric forms and on Hamilton's quaternions. The object of the appendix is to show the power of the authors' vector analysis by using it to set up the Grassmannian and Hamiltonian systems. There is apparently the further object to set forth these two mathematical disciplines in such a way that mathematicians in general, and in particular those mathematicians who think they know something about the systems, shall be led to conceive or reconceive, as the case may be, these systems as they should be conceived. We have no exceptions to take to the authors' presentation of the subject; it is compelling.

There is one remark, found on page 201, which deserves

* Under the title "The unification of vectorial notations," BULLETIN, vol. 16, pp. 415–436 (May, 1910).