

coordinates is also extremely well presented. In the chapter on trilinear coordinates the symbolic notation of Clebsch would be much briefer than the clumsy multiple summations, and fully as intelligible to most readers. Since the author writes down the simultaneous invariant of a conic and a line, it would seem worth while to point out that for variable coordinates u_1, u_2, u_3 this is merely the line equation of the conic.

It is somewhat disappointing that some of the features which make Salmon's Conic Sections a valued companion are not developed. There is nothing on invariants and covariants of systems of conics which are the subject of Salmon's most fruitful chapter. The theory of reciprocal polars is not very fully developed and such ideas as radius of curvature, evolutes, etc., are not introduced.

Part II. on surfaces is developed along similar lines. The most interesting features are probably the treatment of polar properties, the early introduction of line coordinates, and above all the clear discussion of the linear complexes arising. The next volume will doubtless contain much more of interest in space geometry.

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Analytische Geometrie der Kegelschnitte. By W. DETTE. Leipzig, Teubner, 1909. vi + 232 pages, with 45 figures.

THIS admirable elementary text on conic sections is worthy of examination by any teacher of that subject. Both in arrangement and in treatment, there are a number of innovations. The book is divided into three parts: the first of 94 pages is devoted entirely to theory and the study of the general equation in the form $ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$; the next 50 pages contain a classified list of over twelve hundred examples illustrating the text; the remaining part of the volume is devoted to answers to, and also suggestions for solving, the examples of part two.

In the text itself, the six chapter headings: the point, the right line, the ellipse, the parabola, the hyperbola, and the determination of a conic through points and lines, promise little different from the old line American text. But on the first page the author introduces the idea of relative "mass numbers" or magnitudes; that is, for any segment of a line AB , we say $AB = -BA$. The author calls AB and BA the relative "mass numbers," their absolute magnitude being the same. As soon

as coordinates are defined, locus problems are solved and the equation of the circle derived before the straight line is mentioned. Oblique axes and polar coordinates are introduced from the start and every theorem or even remark which applies to oblique as well as rectangular coordinates is starred so as to attract attention. Transformations of coordinates are introduced only as needed. In common with most German texts the idea of harmonic points and lines is introduced early and kept in the foreground.

The fundamental division of curves represented by equations of the second degree is based on whether $ac - b^2 \cong 0$, although the equation of the ellipse is first derived from its focal property. In deriving the polar equation, p , the "parameter," is defined as the focal ordinate, hence taking the vertex as the origin we have the test $y^2 < 2px$ for the ellipse, $y^2 = 2px$ for the parabola, etc. The parabola is defined as an ellipse with one focus at infinity and all properties are derived from this definition, although it is later pointed out that it is equally well the limiting case of the hyperbola.

But perhaps the most striking feature in a book of such an elementary character is the extent to which the pole and polar idea is used. The entire theory of directrices, conjugate diameters, centers, tangents, and axes is based on this in a most attractive way. The "polarized" forms, i. e., $U_{11} = ax_1^2 + 2bx_1y_1 + cy_1^2 + 2dx_1 + 2ey_1 + f = 0$, $U_{12} = ax_1x_2 +$, etc., are regarded as fundamental quantities, and conditions are stated in terms of them; e. g., if $U_{11} \cong 0$, the point P_1 is outside, on, or within the curve; if $U_{12}^2 - U_{11}U_{22} = 0$, the line P_1P_2 is a tangent; if $U_{12} = 0$, the line P_1P_2 is harmonically divided by its intersections with the conic, and so on. While this is not new, it certainly is used more extensively than in any other elementary text.

The last chapter in the book is devoted largely to the theorems of Plücker, Pascal, and Brianchon. It must be very evident to the reader that the few pages of text contain most of the essentials of conic sections, presented in a most modern fashion. The author states that his only reasons for treating each of the conics separately are aesthetic and pedagogic.

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