in accord with the usage for real numbers, so that if the rational numbers a, b correspond to the p-adic numbers $[a]$, $[b]$, we may have $a > b$, $[a] < [b]$. In setting up this correspondence, I have introduced the term monomial p-adic number. On page 130, Hensel assumes that the equation for α is irreducible in $K(p)$. Although not stated explicitly, this assumption underlies §§ 3–7 of the same chapter. In the present account I have therefore avoided this assumption and proceeded at once with the general case; see (6) above.

In addition to the intrinsic interest attached to the new fields or domains introduced by Hensel, his theory has proved to be of such importance in the difficult problems relating to discriminants that it must be granted a permanent footing in the theory of algebraic numbers.

L. E. Dickson.

University of Chicago,
May 9, 1910.

SHORTER NOTICES.

Factor Table for the First Ten Millions. By D. N. Lehmer. Washington, D. C., Carnegie Institution of Washington, 1909. xiv + 476 pp.

The publication of Lehmer's factor table marks an event of the greatest importance in the science of higher arithmetic. The chief factor tables published hitherto are the following: For the first, second and third millions, Burckhardt (Paris, 1817, 1814, 1816); fourth, fifth and sixth millions, Glaisher (London, 1879, 1880, 1883); seventh, eighth and ninth millions, Dase and Rosenberg (Hamburg, 1862, 1863, 1865). Rosenberg's manuscript for the tenth million was presented by his widow to the Berlin Academy of Sciences, but has disappeared. Crelle's manuscript for the third, fourth, and fifth millions was turned over to the Berlin Academy but was found to be too inaccurate for publication. Kulik's manuscripts, placed in charge of the Vienna Royal Academy in 1867 (see Encyklopädie der Mathematischen Wissenschaften, volume I, page 951; *Wiener Berichte,* volume 53, page 460) purport to give the smallest factor of all numbers up to one hundred million which are not divisible by 2, 3, or 5. In Kulik's manuscript each prime not exceeding 163 is represented by a single character (letter or digit), and the higher primes by two characters. Although his scheme effects a decided abbreviation, it increases greatly the difficulty of comparing his manuscript with other tables. Kulik's manuscript is not accurate enough for publication; his tenth million was translated into the ordinary notation by Lehmer and found to contain 226 erroneous entries. Lehmer gives a list of the 246 errors found in the tables by Burckhardt, Glaisher, Dase and Rosenberg for the first nine millions.

The publication of Lehmer's factor table is timely in view of the difficulty of obtaining copies of certain of the earlier tables. It is now practicable for each arithmetician to have at hand, ready for instant use, a factor table in a single volume extending to ten million.

However, the most important question in the comparison of two factor tables is their relative accuracy. When a computer uses a table giving the values of a continuous function, his result should be approximately correct; any considerable error may be laid at his own door. But when an arithmetician relies on a factor table for the primality of a given number, he is entirely dependent upon the accuracy of the table; the entry is either exactly right or wholly wrong, — there is no question of approximation. The independent verification that a proposed large number is actually prime usually entails great labor.

It is therefore in place to inquire into the grounds for the belief that Lehmer's factor table is more accurate than the earlier tables. The sheets (13 by 25 inches) of the corrected typewritten copy of the table were photographed on glass, reduced in size to 12 by 16 inches. Photographic proofs were then corrected by the author. The corrected photographs were transferred to zinc plates, on which any necessary corrections were made. It is believed that this reproduction by photography instead of by movable types has eliminated several sources of error in construction of the earlier tables. Mention should also be made of certain new devices employed in the actual construction of the present table, which tended towards increased accuracy. A certain modification enabled Lehmer to employ stencils only only one-fourth of the length of Glaisher's. Consequently, Lehmer was able to employ the stencil method throughout, whereas Glaisher was compelled to have resort to the less accurate "multiple method" for primes higher than 307. Again, Lehmer's stencil possessed a certain symmetry which was utilized to check its accuracy. Finally, Lehmer employed the improved plan of glueing the successive pages top to bottom in a continuous sheet, which was mounted upon a long bench with a roller at each end.

However, the claim for the greater accuracy of Lehmer's table does not rest so much upon these improvements in its construction as upon the fact that he was in a position to eliminate errors from the proof sheets by noting discrepancies with the earlier tables, as detected by a comparison entry for entry, a comparison made at least five times.

In Kulik's table the rate of error was approximately 1 in 1000 entries; in the other tables used for comparison the rate was lower. There is about an even chance that two computers working independently and with a rate as high as Kulik's will both make an error in the same place in a table extending to ten million. The probability that the same error will occur is therefore negligible.

An error in the case of a composite number is less likely to arise and is of less importance than in the case of a prime, as it would be detected at once by the person using the table. Hence the value of the check afforded by a count of the primes given in a table and its comparison with the number computed by Bertelsen (*Acta Mathematica*, volume 17) by a modification of Meissel's method (*Mathematische Annalen*, volumes 2, 21, 25). In a letter to the reviewer, Lehmer states that the actual count of the primes (including unity) in the first ten millions was found to be 664,580, which agrees with Bertelsen's computed number (increased by one to count unity), and that he has finished about three-fifths of a separate table of the primes less than ten million. In the latter work he checks each successive thousand with Glaisher's count and each successive fifty thousand with Bertelsen's computed number. No discrepancies with the latter have as yet appeared, while the frequent differences with Glaisher's count are all due to the presence of detected errors in the tables from which Glaisher made his count.

While the wide circle of mathematicians and amateurs interested in the theory of numbers is already under the greatest obligations to the Carnegie Institution of Washington for its aid to Lehmer in the construction of his factor table and for its publication, it will be fully satisfied only upon the completion of this monumental project by the publication of a separate table of primes. L. E. DICKSON.