SHORTER NOTICES.

La Geometria non-euclidea, esposizione storico-critica del suo Sviluppo. Da ROBERTO BONOLA. Bologna, Zanichelli, 1906. vi + 213 pp.

Wissenschaft und Hypothese IV: *Die nichteuklidische Geometrie, historisch-kritische Darstellung ihrer Entwicklung.* Von ROBERTO BONOLA, Professor an der Scuola Normale zu Pavia. Autorisierte deutsche Ausgabe besorgt von Prof. Dr. HEINRICH LIEBMANN. Leipzig, Teubner, 1908. viii + 244 pp.

IN the development of the sober science of mathematics a certain dramatic and even sensational element has been furnished by non-euclidean geometry, the history of which is therefore unusually interesting. By its very nature this subject lends itself easily to a historical and critical treatment, like that of the admirable book under review. Unfortunately, the very fascination of the subject has apparently retarded its growth along the substantial lines of actual detailed knowledge: the tendency has been to regard it as a curious and elegant plaything, rather than as the valuable adjunct to euclidean geometry, which it undoubtedly is. The slowness of its growth is illustrated by the fact that although more than eighty years have elapsed since Lobachevsky published his first epoch-making researches, it is only recently that quadric surfaces in non-euclidean space have been carefully studied and classified.

This and most other recent investigations are not mentioned by Bonola. Indeed, as he himself states, the character of his book is distinctly elementary. He begins by taking the reader back to the early period of questioning and doubt as to Euclid's fifth postulate, then carries him through the storm and stress period of creation by Gauss, Lobachevsky, and Bolyai, and is finally content to land him safely in the harbor of modern thought, where projective geometry, differential geometry, and the theory of continuous groups all afford cumulative evidence of the validity of the new doctrine.

In Chapter 5 the author gives several well-known methods of representing or imaging a non-euclidean space in a euclidean space, but omits to mention one introduced by Klein and oincaré and used successfully by Weber and Wellstein in their Encyklopädie der Elementar-Geometrie and by Liebmann (the translator of Bonola's work into German) in his own Nichteuklidische Geometrie, namely, one in which non-euclidean lines and planes are represented by euclidean circles and spheres, respectively.

Two supplementary chapters, one on non-euclidean statics and one on Clifford parallels, Clifford surfaces, and the Clifford-Klein problem, and in the German edition another supplementary chapter on the construction of Lobachevsky parallels, add considerably to the value of the book. There is an index of authors cited, but no general index.

It would be fortunate if we could have an English translation of so valuable and interesting a work; for in English there is nothing covering even approximately the same ground except possibly the scattered papers of G. B. Halsted.

ARTHUR RANUM.

Analytic Geometry. Revised Edition. By E. W. NICHOLS. D. C. Heath and Company, 1908. xi + 282 pp.

THE general scope of this book is the same as that of the first edition which appeared fifteen years earlier. In the first edition the last chapter—a discussion of surfaces—was written by Professor A. L. Nelson and in the new edition this chapter has been entirely rewritten. Otherwise comparatively few changes in the subject matter have been made. The revised edition is very neatly bound in flexible covers—the style so largely used by D. C. Heath and Company lately. The printing, too, is distinctly better than in the former edition.

"The aim of the author has been to prepare a work for beginners, and at the same time to make it sufficiently comprehensive for the requirements of the usual undergraduate course." The first part of this aim has been more successfully carried out than the second. The book is written clearly and contains numerous, well-chosen problems. The conventional order of topics is followed—the conic sections being discussed separately with little emphasis upon their relation to each other. Probably the book is more elementary than would be acceptable in the best engineering schools.

G. H. SCOTT.

Complete Arithmetic. By GEORGE WENTWORTH and DAVID EUGENE SMITH. Boston, Ginn and Company, 1909. v + 474 pp.

THIS book preserves and combines most of the strong features of two well-known series of arithmetics—the Wentworth and