It carries the work in subjects like the roots so far that the ordinary Rechenmeister could not have used it. Moreover, it is written in Latin and is much more extended than the work of Gemma Frisius, so that it appealed neither to the business school nor to the ordinary classical school. A great deal of attention is given to exchange, the rule of three, and the extracting of roots of high order. Attention is also given to problems which would now form part of algebra, and there is a brief treatment of geometry from the standpoint of mensuration.

"While Scheubel is not much appreciated to-day, he was really ahead of his time. He tried to banish the expression 'rule of three' and to substitute 'rule of proportion.' His explanation of square root is in some respects the best of the century, and he dismisses with mere mention the 'duplatio' and 'mediatio' of his contemporaries. He extracts various roots as far as the 24th, finding the binomial coefficients by means of the Pascal triangle a century before Pascal made the device famous."

As to its usefulness, this is a work which no bibliographer of rare books will fail to consult. It will become an authoritative source for writers of mathematical history and the standard reference book on sixteenth century arithmetic for scholars in mathematics everywhere. It would be wasteful of the reviewer's space to speak of the author, because his special fitness is known to practically every student of the history of mathematics, and his scholarship stamps with authority all of his productions.

LAMBERT L. JACKSON.

Coordinate Geometry. By HENRY BURCHARD FINE and HENRY DALLAS THOMPSON. New York, The Macmillan Company, 1909. 8vo. 8 + 300 pp.

IT was generally considered by the writers of the earlier American text-books on analytical geometry and by those who then taught the subject that the material for a first course consisted of the chief metrical properties of the separate species of conic sections. There is a marked similarity between the text in these books and the easier portions of Chapters I, II, VI, X, XI, XII of Salmon's Treatise on Conic Sections (edition of 1869). Within recent years, however, there has been a marked tendency among some of the teachers to regard the acquisition of these isolated facts about parabolas, ellipses, and hyperbolas as a means instead of an end. They are eager to infuse into even a first course some of the spirit of the so-called "modern" geometry and to replace a few of the less important facts of the old course by some of the simplest of the "new" ideas. Many of the recent texts have reflected more or less of this tendency. The advocates of the older method argue that it is necessary to master their facts before the newer ideas can be grasped. They also urge that their material, if thoroughly learned, will occupy all the time allotted; and that any attempt to introduce additional ideas will only result in lack of mastery and consequent confusion. The leaders of the innovation reply that the natural result of progress in mathematics, as in other subjects, is gradually to replace the less important old facts by the more important new ones. They also insist that some of the new ideas will tend to unify the mass of metrical facts.

In looking over a new book on analytical geometry, it is natural, therefore, to inquire whether there is any departure from the conventional body of material and whether there is any striking feature in the method of presentation. There was a strong tendency for some years to introduce into the elementary texts on analytical geometry, as well as those on calculus, many more or less complicated problems from various subjects, especially from physics and statistics. The discussion of loci and graphs gave an excuse for prolonged excursions into domains bordering on the mathematical territory. That a moderate use of such problems is stimulating to thought and is necessary to give some idea of the practical applications of mathematics is probably not often questioned. However, in some cases at least, this practice was undoubtedly overdone. In the present text almost the opposite extreme is found. There are practically no problems showing the applications of mathematics. In Chapter XI, on equations and graphs of certain curves, the path of a projectile is discussed. Perhaps there may be other applications, but they were not apparent in a first survey. There are places where such problems and illustrations are particularly useful. For example, in the first chapter, on coordinates, some illustrations of the use of coordinates and coordinate paper in the charts employed in many businesses to-day would add life to the chapter, without in any way lessening its dignity.

The authors state that it is in deference to usage that the chapter following that on the straight line is devoted to the circle. They urge that it would be better to omit this until the chapters on the parabola and ellipse have been studied. Their reason is that thus "the student sooner realizes the power of the method of the coordinate geometry through seeing it employed in investigating *new* material." On the other hand, in justification of the retention of a short chapter on the circle in this place, it may be argued that, as the *method* is new, the student must gain some facility in its use by employing it on old material before he can have sufficient mastery of it and confidence in it to enable him to use it easily and naturally on the new material. In the treatment of the conics, either of two methods is possible. According to one, the discussion of conics in general precedes the briefer consideration of the particular properties of the different species. In the other method the order of presentation is reversed. There are, of course, arguments for and against each method. The second method is that employed in the present book, where Chapters IV, V, and VI deal with the parabola, ellipse, and hyperbola respectively, while conics in general are not discussed till Chapter VIII.

The treatment of tangents and poles and polars is always an interesting subject for discussion. One criticism that might be made on this book is that there is too much repetition in the subject of tangents. In the cases of the parabola and ellipse, three methods for finding the equation of the tangent in terms of the coordinates of the point of contact are worked out in detail. For the hyperbola the corresponding equation is derived from that for the ellipse. Would it not have been wiser for the authors to have selected one of these methods and omitted the other two? The teacher whose chief interest is in engineering students would probably wish the calculus introduced in the derivation of the tangent to the first conic. This, of course, brings up the ever old and ever new question of whether analytics shall be treated without the aid of calculus, or whether an attempt shall be made to combine the two subjects. As the present text-book is evidently intended for students who expect to have a course in calculus later, there is little use made of its symbols here. Nevertheless, they are not entirely neglected. In one of the chapters on solid geometry, after the equation of the tangent plane to a conicoid in terms of the coordinates of the point of contact has been obtained, the statement is made that the coefficients may be represented by the partial derivative symbols (provided the variables are replaced by the coordinates of the point of contact). Again Table C at the end of the book is a one page explanation of derivatives and partial derivatives.

Poles and polars are not discussed in the separate chapters on the circle, parabola, ellipse, and hyperbola, nor in the chapter on the general equation of the second degree. They appear for the first time in Chapter IX, on tangents and polars of the conic. After the equation of the tangent to any conic in terms of the coordinates x', y' of the point of contact has been obtained, the statement is made that the straight line represented by this equation is called the polar of the point (x', y') with reference to that conic, whether point (x', y') lies on the conic or not. For the circle, it is shown that if the polar of P_1 contains P_2 then the polar of P_2 contains P_1. A solution is given for finding the pole of a given line with reference to a circle. It is also noted that the polar of a point outside a circle is the chord of contact for the tangents from that point. The corresponding fact if the point is inside the circle is stated. No mention is made of harmonic properties. It is to be noted that poles and polars hold a very insignificant place in the plan of this book.

In view of the recent discussions on the question of the place of loci in college entrance examinations, it is interesting to notice the treatment of this subject in an elementary text intended for the early part of a college course. In addition to the illustrations of loci found in the several conics and in their diameters and in certain of the special curves, the last chapter in the plane geometry is devoted to this subject. Nine examples are fully explained and then, after some general remarks on loci, fifty carefully graded problems are given. The idea of loci is certainly made prominent.

Solid geometry occupies more than one third of the volume. This is a more extended treatment than is usually found in the elementary texts. In the present book the first two chapters, on coordinates and direction cosines and on planes and straight lines, are very similar to corresponding chapters in C. Smith's Elementary Treatise on Solid Geometry. These two chapters occupy slightly less than one half of this part of the book. After a discussion of the general shape and the sections of the different species of conicoids and a brief survey of polar coordinates and transformation of coordinates, a chapter is devoted to the general equation of the second degree. Centers and diametral and principal planes and the classification of conicoids are then briefly treated. There is a short discussion of the invariants of the general equation under a transformation from one orthogonal system of axes to another orthogonal system.

The text proper is followed by six short tables, which deal with algebraic and trigonometric formulas, derivatives and partial derivatives, four-place table of logarithms of a few numbers, lengths of arcs in radians, and the letters of the Greek alphabet. The book is concluded by a set of nine very good plates showing the silk thread figures of the ruled surfaces of the second order, as well as the usual plaster models of the conicoids. Indeed, one of the best features of the book is to be found in the excellence of the numerous figures. The young man in whose hands this text is placed will probably note first of all that it is small enough to fit into his pocket. By employing rather thin backs and paper that is not too heavy and by lessening the margins, the size and weight of the volume have been reduced to a minimum. Perhaps the strongest feature of the book is to be found in the abundant supply of examples. After each bit of theory there are some exercises, and at the end of each of the longer chapters there is a set of about fifty carefully graded problems.

E. B. COWLEY.

Leçons sur les Fonctions définies par les Equations différentielles du premier Ordre. Par PIERRE BOUTROUX. Paris, Gauthier-Villars, 1908. 190 pp.

THE little volume bearing the above title is one of the series of monographs on the theory of functions published under the editorship of E. Borel. The author's aim is to set forth the theory of functions defined by a differential equation as based on the work of Painlevé. He abandons the "local point of view" of Cauchy and studies the ensemble and form of the integral not only in the neighborhood of a point but in general. The particular question discussed is one raised by Painlevé, viz., how does the solution behave when the initial point x_0 at which it is considered varies from point to point in an arbitrary manner.

The book is divided into five chapters. Chapter I presents the fundamental notions. After a review of the usual theory of singular points the following theorem of Painlevé's is dem-