THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE sixteenth regular meeting of the San Francisco Section of the American Mathematical Society was held at the University of California, Saturday, September 25, 1909. The following members were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor A. O. Leuschner, Professor H. C. Moreno, Professor C. A. Noble, Mr. E. W. Ponzer, Mr. H. W. Stager, Professor A. L. Whitney.

The following officers were elected for the ensuing year: Professor Blichfeldt, Chairman; Professor Noble, Secretary; Professors Hoskins, Lehmer, Noble, Program Committee. The next two meetings are to be held respectively at Stanford University February 26, 1910, and at the University of California September 24, 1910.

The following papers were read at this meeting:

(1) Professor D. N. LEHMER: "On the arithmetical theory of pencils of binary quadratic forms" (preliminary communication).

(2) Professor H. F. BLICHFELDT: "On the infinitude of primes in certain arithmetical progressions" (preliminary communication).

(3) Mr. G. F. MCEWEN: "The motion of a viscous fluid between two parallel planes" (preliminary communication).

(4) Professor L. M. HOSKINS: "The strain of a gravitating compressible elastic sphere."

(5) Professor A. O. LEUSCHNER: "An equation giving the geocentric distance in the problem of determining parabolic orbits from geocentric observations."

Mr. McEwen was introduced by Professor Blichfeldt.

Abstracts of the papers are given below in order as numbered in the foregoing list:

1. Professor Lehmer gives a study of the arithmetical properties of the forms included in the pencil $\alpha A + \beta B$, where α and β are integers and A and B binary quadratic forms. The

forms of the pencil must have determinants representable by a certain binary quadratic form $H = Dx^2 + \Theta xy + D'y^2$, where D and D' are the determinants of A and B and Θ is the joint invariant. If $A = (abc)$, $B = (a'b'c')$, the form $J = (ab' - a'b)x^2 + (ac' - a'c)xy + (bc' - b'c)y^2$ is also of fundamental importance in the theory. A pencil may be found having a given form J. A pencil may or may not be found having a given form H, according as H is or is not of the principal genus. The form H is the duplicate of J if J is a primitive form.

2. By elementary algebraic processes, involving the approximate evaluation of certain factorials, Professor Blichfeldt proved that the arithmetical progressions k, $11 + k$, $22 + k$, $33 + k$, $\cdots$ (k prime to 11) contain an infinite number of primes each.

3. Mr. McEwen's paper is in abstract as follows: A viscous fluid is confined between two parallel planes, one being fixed, the other having a displacement in its own plane,

$$x_1 = ae^{-\alpha t} \sin \sigma t.$$

Assuming the distance between the planes to be great, the displacement of the fluid at the distance y from the moving plane is

$$x_2 = ae^{-\alpha t - \beta y} \sin (\sigma t - \beta y),$$

where $\beta = \sqrt{\sigma\rho/2\mu}$, ρ = density of the fluid, μ = the coefficient of viscosity of the fluid.

A gravity pendulum is hung so that a small plane attached to its lower end is parallel to the plane of vibration of the pendulum and the fluid in which it is immersed. x_3 is the displacement of this plane. x_1', x_2', and x_3' are the maximum values of x_1, x_2, and x_3.

$$\frac{x_2'}{x_1'} = e^{-\beta\left(1+\frac{\alpha}{\sigma}\right)y} \quad \text{and} \quad \frac{x_3'}{x_1'} = \left(1 - \frac{\alpha^2}{2\sigma^2}\right) e^{-\beta\left(1+\frac{\alpha}{\sigma}\right)y},$$

if $x_3 = ae^{-\alpha t} \sin \sigma t$ when the plane is not in the fluid and if α/σ is small.

4. In Kelvin's well-known solution of the problem of the strain of an elastic sphere, the bodily forces are assumed to be known functions of the coordinates of position. When self-

gravitation is considered this solution is inapplicable, except in the case of incompressibility, because the force of attraction acting upon any volume element depends in part upon the change of density produced in that element by the strain and upon the change of density distribution of the attracting mass. A solution of the problem taking account of the actual gravitational forces in the strained configuration is given in Professor Hoskins's paper. The problem is worked out completely for the case in which the strain is due to disturbing forces of the type of tidal or centrifugal forces, and numerical results have been obtained corresponding to several different values of the ratio of the elastic constants. The strain at any distance from the center being specified by two quantities — the ellipticity of the originally spherical surface and the angular displacement of a radius vector inclined 45° to the axis of symmetry — it is found that for a given value of the rigidity modulus, the former of these quantities is decreased and the latter increased by compressibility. The solution has also been generalized so as to apply to the case in which the potential of the disturbing forces is any spherical harmonic of degree not less than 2.

5. In his adaptation of the "short method of determining orbits" to the direct computation of a parabola for comets, Professor Leuschner derives the geocentric distance ρ at a fixed date from the equation

$$(z-p')^2 - \frac{n}{[(z-c)^2+s^2]^{\frac{1}{2}}} - q'^2 = 0,$$

where $z = \rho/R$, and R is the distance of the sun. p', n, q'^2, c, and s are auxiliary quantities depending on observed coordinates and other data. c and s are the sine and cosine of the angle ψ subtended at the observer by the arc between the comet and the sun.

The solutions are given by the intersections of the parabola $y = z'^2$ and the curve $y = n/\sqrt{[z'-c']^2 + s^2} - q'^2$ for which z' is real and positive, where $z' = z - p'$ and $c' = c - p'$.

There will be either one or three solutions. Three solutions exist, if

either

$$p' > 0, \quad c > 0;$$

or

$$p' > 0, \quad c < 0, \quad 90° < \psi < 125°\ 16';$$

or

$$p' < 0, \quad c > 0, \quad 0° < \psi < 54° \, 44',$$

and if

$$\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2 < 0,$$

where

$$\frac{p}{3} = \frac{1}{81}\,[9(2s^2 + q'^2) - 7c'^2]\,; \quad \frac{q}{2} = \frac{5c'}{9}\left[\frac{p}{3} + \frac{1}{9}\left(m^2 - \frac{11}{10}q'^2\right)\right]$$

and

$$m^2 = c'^2 + s^2.$$

For solution the equation in z is written in the form

$$y' = (\zeta + c')^2 - (\eta - q'^2) = f(\vartheta) = 0,$$

where

$$\zeta = s \tan \vartheta\,; \quad \eta = \frac{n}{s} \cos \vartheta.$$

A convenient graphical solution is proposed for the solution of $f(\vartheta) = 0$. Then

$$\rho / R = z = s \tan \vartheta + c.$$

Geocentric distances correct to four or five decimals result from the graphical solution. Further decimals may be obtained by a simple differential correction.

In practice no case with three solutions has been encountered.

C. A. NOBLE,
Secretary of the Section.

THE WINNIPEG MEETING OF THE BRITISH ASSOCIATION.

THE seventy-ninth annual meeting of the British Association for the advancement of science was held in Winnipeg August 25 to September 1. Fourteen hundred members and associates were in attendance. The opening event was the address of the President of the Association, Sir J. J. Thomson, on Wednesday evening, August 25, in which he gave an account of some of the more recent developments in physics and in his opening remarks took occasion to urge a closer union between mathematics and physics and to emphasize the advantages of