it these additive convergent aggregates, which may also be called irrational numbers. It is then proved that, in this enlarged domain, it is possible to extract the nth root of any natural number, and to represent the ratio of two lines, whether they are commensurable or incommensurable.

The subject matter of the rest of the volume may perhaps be sufficiently evident from the general headings of the last three lectures. They read as follows : Additive aggregates of an infinity of positive and negative rational numbers ; additive aggregates of an infinity of complex numbers of the form $a + bi$; multiplicative aggregates of an infinity of numbers. The value of the volume is greatly enhanced by illustrative examples, and it may be heartily recommended even to those who are just beginning graduate work in our universities. It need scarcely be added that a clear comprehension of this theory of irrational numbers will clear up many difficulties as regards the theory of absolutely convergent series with numerical terms.

G. A. MILLER.

Magic Squares and Cubes. By W. S. ANDREWS. With Chapters by PAUL CARUS, L. S. FRIERSON, C. A. BROWNE, JR., and an Introduction by PAUL CARUS : Chicago, The Open Court Publishing Company, 1908. vi + 199 pp.

Among the Arabians magic squares were known in the ninth century of our era and about this time they played an important rôle in Arabian astrology. A special work on the subject is attributed to an Arabian mathematician named Tâbit ben Korrah who died in 901,* and H. Suter mentions several other early Arabian writers on this subject in his work entitled Die Mathematiker und Astronomen der Araber und ihre Werke. These facts are not in accord with the statement on page 1 of the book under review, which reads as follows : " The earliest record of a magic square is found in Chinese literature dated about 1125 A. D."

The present work is, in the main, a direct reprint of articles which appeared in the *Monist* during recent years. Its author is an electrical engineer who, during his leisure hours, " has given considerable thought to the working out in his own original way the construction of magic squares and cubes of various styles and sizes." As may be inferred from this excerpt

* Encyclopédie des Sciences mathématiques, t. 1, vol. 3 (1906), p. 63.

from the announcement and also from the preceding paragraph, the author made little use of the extensive literature on the subject, but has aimed to give a clear and interesting account of the results to which his own labors and those of his correspondents have led. An important exception is furnished by the chapter on the "Franklin squares," which gives a very interesting account of magic squares constructed by Benjamin Franklin, including a letter in which Franklin says "I make no question, but you will readily allow the square of 16 to be the most magically magical of any magic square ever made by any magician."

The general headings of the various parts of the book are as follows: Introduction by Paul Carus, magic squares, magic cubes, the Franklin squares, reflections on magic squares by Paul Carus, a mathematical study of magic squares by L. S. Frierson, magic squares and Pythagorean numbers by C. A. Browne, some curious magic squares and combinations, notes on the various constructive plans by which magic squares may be classified, and the mathematical value of magic squares. Under the sub-heading "Mr. Browne's square and lusus numerorum" Paul Carus gives instances of numbers which exhibit surprising qualities without being in the form of a magic square.

In 1896 Emory McClintock read a paper before the American Mathematical Society entitled "On the most perfect forms of magic squares with methods of their construction" in which he introduced the term pandiagonal magic squares for a type of squares which were called diabolic by Lucas and are generally known in Europe by the latter term. The paper by McClintock was published in the *American Journal of Mathematics*, volume 19, and constitutes one of the most important American contributions to the subject. Extensive bibliographical data on this subject may be found in volume 1 of the Subject Index of the Royal Society of London Catalogue of Scientific Papers, pages 84 and 85; in Ahrens's "Mathematische Unterhaltung und Spiele," 1901; and in the "Encyclopédie des Sciences mathématiques," tome 1, volume 3, pages 62 to 75. It is of interest to observe that the French edition of the great mathematical encyclopedia devotes thirteen pages to this subject while less than a page is devoted to it in the German edition.

As has been observed above, the few historical references in the present work should not be taken seriously, but in other respects it can be commended highly to those who are attracted by marvellous relations among natural numbers. The author is looking forward to a second edition in which a number of slight errors will be corrected, and he has had the courtesy to send the reviewer a marked copy in which the following changes are suggested: The term "perfect square" as used on page 2 is replaced by "regular square." In the second and third lines from the bottom of page 5 "twenty-eight" and "sixteen" are replaced by twenty and eight respectively, and in the first line of page 6 "twelve" is replaced by sixteen. The term "prime number" as used on page 14 and in many other places in the book is replaced by primary number. In the last line on page 65 the expression "first and last" should read last and first. Near the middle of page 179 the statement marked I. should be followed by "with four exceptions." These errors are, however, not sufficiently serious to detract much from the value of the volume.

G. A. MILLER.

Exercices et Leçons d'Analyse. By R. D'ADHÉMAR. Paris, Gauthier Villars, 1908. 208 pp.

THE subtitle of this volume is "Quadratures, équations différentielles, équations intégrales de M. Fredhoim et de M. Volterra, équations aux dérivées partielles du second ordre." It will be seen that the topics treated are thoroughly up to date. The book is meant, as the author says in his preface, to supplement the larger Traités and Cours. An introduction of 22 pages presents a brief statement of some of the principal theorems on differential geometry and analysis, together with references for their proofs and for further developments. Then follow chapters on quadratures; the functions of Legendre, Bessel, Euler, etc.; partial differential equations of the elliptic type, including a brief treatment of Fredholm's integral equation; equations of the hyperbolic and parabolic types; and two chapters on miscellaneous problems. The book, in spite of its decidedly fragmentary character, will prove useful both by furnishing a source of interesting problems and by giving the reader at least a superficial idea of many recent developments in analysis. The indications given as to the scope and purport of theorems mentioned (to say nothing of their proofs) are, however, frequently so meagre that the reader seeking to gain information will often be in doubt as to how they should be