him what color is to Sorolla or form to Michelangelo or rhythm to a reader of Poe, and the teacher is a poor one who does not appreciate this fact. It is because mathematics is itself a subject full of interesting situations, of wonder, and of rhythm that more students enjoy it than some of our pedagogical agitators think, and it is because of this that much of the effort to humanize mathematics to-day is really, though well meant, an effort to make it less human.

It is partly for those mistaken teachers who feel that mathematics has not the same interest per se that music or art or literature has, that Mr. Ball prepared this interesting collection upwards of seventeen years ago. But it was also for the mathematician himself, who abuses his nerve system in his love for the more serious side of the science, that the book was written, even as Punch is published not merely for the casual reader but also for the statesman who needs to see his labors in a different light after a night in parliament.

The English work has now passed through four editions and the French translation through two, which testifies anew to the pleasant style and to the wisdom of selection that characterizes Mr. Ball's various publications.

The French edition is considerably more extended than the English original in some respects, filling three volumes. The chief departure from the original is in Chapter I, Some arithmetical questions. This occupied less than forty pages in the English edition, but it makes up the first volume, of over three hundred pages, in the translation. The added material relates largely to the history of numbers and to interesting problems of early and medieval times. The mysticism of numbers, so exhaustively treated from the religious side by Bungus three centuries ago, speculations on the platonic number, curious properties of decimal numbers, the application of algebra to number games, and the elementary theory of numbers in general are some of the features of the French edition that make it well worth placing upon the shelves of any mathematical or general library.

DAVID EUGENE SMITH.

Lehrbuch der Kristalloptik. Von F. POCKELS. Teubners Sammlung XIX. Leipzig, B. G. Teubner, 1906. x + 520 pp.

ALTHOUGH the past five years have seen the publication of a large number of books on optics, the subject is so broad and

may be treated from so many points of view that there has been relatively little duplication in the different works and there is still room for other treatments. Where one book may lay the stress on the analytical theory and be replete with formulas, another may take the physical side and with relatively few formulas establish the chief points of relation between optics and electromagnetism, and yet another may deal with optics from what might be called a dynamical point of view. Pockels's book is none of these, but as its name indicates is concerned with crystalline optics; in fact it will offer much more of interest to the crystallographer than to the mathematician. Yet so thoroughly has the author covered his field and so encyclopedic is his treatment, that any student of optics, whether primarily interested in crystals or not, will find frequent occasion to consult the work, if only to ascertain what may be the known experimental facts with regard to the subject.

After a short introduction, the author divides his work into four major parts which treat respectively transparent crystals without rotary properties, crystals with rotary properties, absorbing crystals, and effects of external influences upon optical properties. Of these divisions the first is naturally the longest and indeed contains more than half of all the matter in the book. From the very introduction the presentation is true to the main object of the author, namely, to describe the phenomena of crystalline optics rather than to construct a well-knit theory. The start is made, not with theories, but with a few statements concerning the propagation of light, the wave surface, rays and plane waves, and Huyghens's principle from which many of the essentials of crystalline optics may be obtained without the need of intricate formulas or detailed physical considerations. Thereupon follow definitions of the light vector, of natural and polarized light, and the presentation of trigonometric and exponential representations of the light vector.

The method of the introduction is pursued during the first two chapters of the first main part of the work. The propagation of light in uniaxial crystals is described with the aid of the wave surface and related surfaces derived from Huyghens's principle. Here, as throughout the book, tables of the optical constants of the type of crystals considered are freely introduced. Even the phenomenon of dispersion with an appropriate table of data is mentioned at this early stage. The second chapter is almost an exact parallel of the first, but treats

biaxial crystals. Thus without any complicated analytical or physical investigations and in the short space of about seventy pages the author has succeeded in giving the reader an excellent account of the general question of the propagation of light in crystals and of the geometric method of discussing the propagation.

The third chapter on the theoretical physical foundations for the theory of light is a model of excellent and concise presentation of a difficult and often confused subject. The trouble is that there are so many different theories of light which give results differing very slightly, usually so slightly that there is no crucial experiment sufficiently accurate to distinguish between them. The elastic theory with its various developments is first mentioned. But it is the electromagnetic theory which naturally comes in for the major part of the discussion. In the propagation of electromagnetic waves there are four vectors, the magnetic induction B, the magnetic force H, the electric induction or displacement D, and the electric force E which may possibly be taken as the light vector. Of these the first two B and H are so nearly parallel in bodies which propagate light that it would be hopeless to distinguish between them and useless to consider as different the theories built upon them. Although many theorists use these vectors, Pockels discards them in favor of the electric vectors, as would be expected in view of results obtained from experiments with stationary waves. As between D and E, the author chooses D as his light vector. He then has a few words to say about the electron theories with especial reference to their bearing on dispersion. Although Pockels thus introduces a little physical optics into his work, the mere fact that Zeeman's name does not occur here or elsewhere is sufficient to show how strictly he adheres to his aim of treating the optics of crystals and how carefully he avoids being led off into the general realm of physical optics.

The fourth chapter is on reflection and refraction. To show the detail with which the subject is presented it will be sufficient to mention the titles of the articles on total reflection. They are: general conditions, methods of observation, the limiting cone of rays in the case of reflection from a uniaxial crystal, polarization of the limiting rays, special cases of total reflection from biaxial crystals, singular phenomena connected with conical refraction, determination of optical constants for

biaxial crystals by means of observations on total reflection. It would scarcely be possible to find a more detailed or simpler presentation of all this material. Many works on analytical or physical optics hardly mention these subjects at all except in the most general way. Several chapters further on in the work, the author takes up the physical basis for these results and develops the formulas from the electromagnetic conditions at the interface of two media.

There is little need of prolonging this review with the recitation of the course of the various chapters. Enough has been said to show the method upon which the author has constructed his book and the detail with which he has written. There is one point in which an improvement might be suggested. The plates which exhibit the elaborate and intricate phenomena of interference are all in black and white. This is a great pity; the beautifully modulated color schemes are the chief attraction of the figures and the author's detailed tables of the colors that are found in some special cases by no means take the place of the actual colors on the plates. If the coloring of the plates had to be done by the eye and hand, there might be good excuses for omitting it; but natural color photography is now so well developed that very good photographs of these effects can be obtained and reproduced.

The student of optics, who frequently finds it very hard to lay his hand upon a large and accurate presentation of the phenomena of crystalline optics, will refer constantly to this work and will feel under deep obligations to its author for the pains taken in preparing it.

E. B. WILSON.

NOTES.

THE July number (volume 10, number 3) of the *Transactions of the American Mathematical Society* contains the following papers: "Projective differential geometry of curved surfaces (fifth memoir)," by E. J. WILCZYNSKI; "On the osculating quartic of a plane curve," by W. W. DENTON; "Note on a system of axioms for geometry," by A. R. SCHWEITZER; "Irreducible homogeneous linear groups in an arbitrary domain," by W. B. FITE; "On the integration of the homogeneous linear difference equation of second order," by W. B. FORD; "On Cantor's theorem concerning the coefficients of a convergent