whose form shows directly that an invariant must be independent of x, y, ρ, and therefore a function of σ only.

If u, x belong to a ray passing through the margin of the image and the axial point of the diaphragm, and v, y similarly to a ray passing through the center of the image and the edge of the diaphragm, a geometric construction shows that the expression $uy - vx$ is equivalent to the product of refractive index, lateral radius of image, and tangent of angular semi-aperture of central pencil. The invariant σ is therefore identical with the expression which occurs in the well-known equation pointed out in a special case by Lagrange, but first found as general by Helmholtz.* The result here shows that this is essentially the only relation expressible in terms of invariants which gives a general property of the paraxial transformation.

CHICAGO. ILL.,
April, 1909.

SHORTER NOTICES.

Analytische Geometrie der Ebene. By C. RUNGE. Leipzig, B. G. Teubner, 1908. 198 pp.

SHALL we regard elementary analytic geometry, the analytic geometry we teach in a first course to freshmen or sophomores, as a body of doctrine with which it is useful for the student to become acquainted, or shall we rather regard it as an instrument with whose use he is to be made so familiar that he shall always be ready to employ it even in a quite new problem? This is a question which every teacher of analytic geometry and every writer of a text-book on the subject is called upon to face. Upon its answer the nature of the text-book written or selected for use will depend. Few persons, it is true, would go to such an extreme as to adopt without qualification either of the views above referred to. Those who regard it as their main object to inculcate a beautiful and important doctrine would deem it essential that the student gain the power of making the application of the general theorems learned to concrete cases, and the teacher who regards elementary analytic geometry primarily as a method whose use is to be taught would not neglect the opportunity of explaining

* Handbuch der physiologischen Optik, 1. Auflage, p. 50.

and illustrating the use of this method, so far as possible, by means of problems which are in themselves of lasting interest and importance. With this reservation, however, the above classification serves to separate teachers and writers of text-books fairly well into two opposing camps.

In the former camp it is customary to take as one of the largest and most essential parts of the body of doctrine to be inculcated the theory of the conic sections; and the subjects of conjugate diameters, poles and polars, the general equation of the second degree, etc., form essential chapters with whose contents the student is supposed to become familiar. On the other hand such a teacher will usually be content, in the main, with numerical problems, which serve to test the pupil's understanding of the formulas and principles developed.

The teacher who adopts the second standpoint will also usually contrive, as has been suggested above, to familiarize his students with many facts concerning conic sections; but he will do this incidentally, his main concern being all the time to train his student to do things for himself. He will, therefore, regard the numerical problem as the lowest and least useful of all problems (except, indeed, for the pupil incapable of rising higher), while the problem which requires the student to handle his instrument in a slightly new way will be regarded as the highest type, to be used sparingly on account of its difficulty, but as invaluable because it gives to the good student the very best training possible in becoming a real master in the use of his method.

There is still a third current which has been running very strongly of late both in this country and in England, which, so far as it is concerned with the subject with which we are here dealing, has as its motto: "Analytic geometry is a necessary evil. Let us have as little of it as possible." The devotees of this cult, who commonly regard all mathematics as merely a tool, naturally take the same point of view with regard to analytic geometry, but, being engineers or in the employ of engineers, they believe that they have no interest in anything beyond the very simplest geometrical relations, and that consequently this particular tool needs only very slight development at their hands. In the rudimentary condition in which they are willing to leave it, numerical problems are about all that one can venture upon, and this is all they ask.

Professor Runge's book has an individuality and interest of its own and refuses to fit in perfectly to the pigeon-holes of any classification. This is just what one familiar with his other writings would expect; and yet some of his statements have at first a very familiar ring. We quote the opening words of the preface:

"This book has grown out of lectures which for many years I was in the habit of delivering at the technological school at Hannover, and in writing it I have had constantly in mind the needs of the engineer. It seems to me that the analytic geometry taught in technological schools should, first of all, be a tool for following up geometrical relations by means of numerical computations."

The programme here laid out is carried through, so far as the simplest properties of the straight line and circle are concerned, in Part II, which extends from page 20 to page 63. This part is perhaps the most distinctive and interesting of the whole book. The problems taken up are of the very simplest types: A line is determined by two points, to compute the ordinates of those points on this line which have given abscissas. Or again: To compute the coordinates of the point of intersection of two lines each of which is determined by a point through which it passes and the angle it makes with the axis of x. Each of these problems (they are the first two, and are typical) is treated at length by several different methods, four or five pages being devoted to each problem. This seems surprising until on closer examination we notice that questions of analytic geometry proper are here quite pushed into the background by pure questions of convenient forms of numerical computation. What method will be best if logarithms are to be used? If we use the slide-rule? If we use a computing machine? How can the numerical work be best arranged? These are all good questions, and we Americans may well wish that systematic courses on numerical computation were more frequent in this country. It seems, however, to the reviewer that such questions should hardly be considered at the very beginning of a course on analytic geometry, as they are then in danger of overshadowing the fundamental principles of the subject.

Perhaps, however, in spite of its title and general appearance, it is hardly fair to treat this book as one from which the beginner will get his first acquaintance with analytic geometry. It should rather be regarded as supplementing in a most interesting way the more traditional treatises from which students will continue to derive most of their working knowledge. Moreover the insistence on the numerical side of the subject, to which we have referred, is only one feature of the book. Long sections of a decidedly theoretical character follow in which affine and perspective transformations of the plane play an important part. Indeed the ellipse, hyperbola, and parabola are introduced as the images of the circle under these transformations, and the whole theory of these curves is made to depend on this point of view. Homogeneous coordinates are also treated at length in the last chapter.

On the whole the book departs less essentially than one would at first suppose from the traditional German text-book in which the subject is presented rather from the point of view of *kennen* than *können*. It is still a body of doctrine which is presented, though the choice of subjects is somewhat unusual, including as it does, besides the subjects already mentioned, a section on the computation of stresses in frameworks of light rods, and an introduction to some of the most elementary aspects of the use of vectors. Even the questions of numerical computation may fairly be regarded as part of this scheme rather than as an attempt to put the student on his own mettle. To a teacher collecting material for a course on numerical computation the section to which reference has just been made will be found most useful. MAXIME BÔCHER.

Gruppen- und Substitutionentheorie. By Dr. EUGEN NETTO. Sammlung Schubert LV. G. J. Göschen, Leipzig, 1908. viii + 175 pp.

IN conformity with the general plan of the Sammlung Schubert Professor Netto aims to give in this book an introduction to the theory of groups of finite order. He has succeeded admirably in his purpose. Those readers who are not already familiar with the details of the theory will find Chapter II particularly valuable in fixing for them the fundamental notions of the subject, if they take the pains to work through the details. Indeed we do not know if there is another place where this particular phase of the subject is treated so happily. But this is by no means the only good chapter. They are all excellent and the book as a whole is a fine example of clear and attractive exposition.

The author introduces some new notation which is of value in the interest of brevity. On page 35 the greatest common sub-