Hermite: "La démonstration de M. Klein est extrêmement intéressante, elle repose entièrement sur la conception des feuilles multiples de Riemann et des considérations géométriques. Cela prouve bien l'utilité de ce mode de représentation, mais je vous dirai que la géométrie y intervient trop, à mon gout. . . . Dès que j'ai vu l'énoncé du théorème je n'ai pu m'empêcher de penser qu'on pourrait y arriver par la méthode de Sturm en s'appuyant sur les relations entre les fonctions F contiguës, etc." Hermite replies "Mais la communication extrêmement intéressante que vous m'avez faite du beau théorème de M. Klein . . . m'arrache à la torpeur qui m'a envahi depuis plusieurs jours. Je vous chicanerai au sujet de votre prétention de le démontrer par la méthode de Sturm . . . Il me semble que cette méthode n'a jamais été appliquée et n'est applicable. . ." Whereupon Stieltjes in his next letter playfully begins to develop his method with: "Maintenant, permettez moi de défendre mon idée de démontrer le théorème de M. Klein à l'aide de la méthode de Sturm." A subject of unfailing interest especially in the later letters is the Γ function in all its astonishing and intricate relations. Mechanical quadrature and asymptotic developments are also favorite topics.

Thus we might go on describing these delightful mathematical causeries which follow one another in rapid and unaffected succession, but we hope that the above will give an idea of the rare treat these letters will afford.

JAMES PIERPONT.

Cartesian Plane Geometry. Part I.: *Analytical Conics.* By C. A. SCOTT, D.Sc. London, J. M. Dent and Company, 1907.

IF this most excellent book is really meant to be a text-book for schools, we are afraid that some one will now have to make a school for the text-book. If it is meant for the libraries of teachers and prospective teachers, it is as near perfection as one could wish; full of the best ways of doing things, of excellent examples and of inspiration for every real teacher. So, whatever faults we may see in the book are those which arise when we consider it as a function of the things it is meant to be as a text-book.

The book, rather unprepossessing in its outward appearance, contains 428 pages, a great number of illustrative examples, and 700 problems. Its thirteen chapters, which of course are a continuous development of the theory of conics, are headed as follows: Introduction, Coordinates of point and line, Representation of point and line by equations, Loci and envelopes, Conics, Relation of straight lines and curves, Tangent at a point and polar properties, Bisected chords and diameters, Asymptotes, Properties of conics (summary), Change of axes, Systems of conics and miscellaneous examples.

The distinctive feature of the book is the use of line coordinates throughout. This, as the introduction states, makes many problems easier, and immensely increases the power of the subject. Geometrically, the line as element and line envelopes are as simple as the point and point loci, and it seems too bad that the unfortunate leaning of our ancestors toward the use of a point to pick their hieroglyphics and draw their pictures, should have led us to be almost void of visualization of how curves must look if their tangents satisfy certain conditions.

Other features of the book are the introduction of the ideal elements by means of ratios defining a point on a line; the use of the circular points and null lines with their relations to foci; the power of a point with respect to a curve; abridged notation; the reality of the imaginary elements; centers of curvature and evolutes; graphical solutions of equations and the determination of conics cutting a circle in a regular polygon. A great deal is said on the subject of bisected chords and diameters and on the properties of configurations arising from conormal points on a conic; in fact the long list of miscellaneous examples worked out in the last chapter has many of these problems which would be the despair of the ordinary undergraduate, *e. g.*, the problem of finding the centroid and orthocenter of a circumscribed triangle of a conic whose points of contact are conormal covers four pages of rather tedious algebra.

Space will permit mention here of but a few of the admirable arrangements and developments of this book.

First of all is the use of line coordinates to solve those problems for which they are best adapted. In this way the student is given another weapon with which to attack the subject; however, Professor Scott often dulls it for him, by the apologetic way in which she uses it, for she usually follows a simple demonstration with the use of line coordinates by a difficult one in point coordinates.

The condition of incidence of a point and line is derived before anything is said about the equation of a point or line, which thus arises naturally. At the beginning of the book various types of coordinates are used, though no examples throughout the entire book are solved by the use of polar coordinates, and thus the student reader will fail to appreciate their power.

A good idea is the derivation in general of the equation of a conic without regard to axes

$$(x - p)^2 + (y - q)^2 = \frac{e^2}{l^2 + m^2}(lx + my + n)^2$$

and hence arrival at once, by a proper choice of axes, which is seen analytically, at simple canonical forms for the different conics.

The general treatment of ideal and imaginary elements on the same basis as those ordinarily called real ones is well illustrated by the treatment of asymptotes as tangents at infinity.

Parametric equations for certain problems in conics and straight lines are used, and although in many instances these are introduced only to be got rid of, yet their importance is emphasized more than in some books. It seems strange that someone does not thoroughly develop this extremely important type of equations, in an elementary text-book.

The line equation of the general conic is introduced very cleverly, as follows: First the equations of the lines through the origin and the points of intersection P_1, P_2 of the conic with the line $\xi x + \eta y + 1 = 0$ are found by means of the necessary condition of homogeneity and the abridged notation; then the condition that these lines through the origin and hence P_1, P_2 be coincident is imposed and thus the condition that $\xi x + \eta y + 1 = 0$ be a tangent is found.

The idea of the graph of an equation seems to be greatly neglected; in fact the whole book gives one the impression that the mere picture of the equation has but slight importance in an analytic geometry. So we find very few examples of point graphs and practically none of line loci. As a natural sequence of this point of view Professor Scott omits the discussion of an equation, as such. This seems like a defect to us, both from the pedagogic and practical standpoint, for in few other ways does such simple analysis yield such good results.

As the book is essentially analytic, all pure geometry examples are put in the background. We cannot agree with this way of presenting the subject to the immature student, for we believe that very few of the latter ever become interested at the beginning in geometry which is only analytic.

The subject of change of axes is rather unsatisfactorily treated. In the first place it is left almost to the end of the book, and thus the student is deprived of the use of a powerful tool for simplifying his problems and for avoiding what Professor Scott designates "algebraic gymnastics." Secondly the formulas of transformation are unrigorously derived, holding necessarily only for the special figures of the book. Finally all examples are worked by comparison with the general case *i. e.*, the general equation of the conic is transformed and its new coefficients found and these are used for the special cases. This undoubtedly gives good practical results; but the student would soon lose sight of what he was doing and the raison d'être of his formulas.

One other fault of the book appears to us to be that as a text-book it contains so many ideas and methods that it would be confusing to any student; *e. g.*, under diameters and reciprocal polars the same examples are worked in detail in three different ways. This makes the book invaluable for our libraries; but if we judge the young student aright, the study of these different methods would tend to confuse what ideas he might successfully gather from the most simple and hence best method of attacking the problem.

E. GORDON BILL.

Grundlagen der Geometrie. Von DAVID HILBERT. Dritte Auflage. Leipzig, Teubner, 1909. vi + 279 pp.

THE present edition of Hilbert's celebrated treatise on geometry appears as a seventh volume in a series of essays, "Wissenschaft und Hypothese," whose first volume is a translation of Poincaré's well known book bearing the latter title. It differs but little from the second edition. Two reprints of papers by Hilbert, "Ueber den Zahlbegriff"* and "Ueber die Grundlagen der Logik und Arithmetik,"† appear as appendices VI and VII respectively; also additional references to investigations of other authors are inserted.

Since very full consideration has been given Hilbert's book in the BULLETIN ‡ and elsewhere § the reviewer must refrain

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* *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 8 (1900).

† Verhandlungen des III Internationalen Kongresses in Heidelberg, 1904.

‡ See vol. 6, p. 287; vol. 9, p. 158; vol. 10, p. 1.

§ Cf. E. B. Wilson, *Archiv der Mathematik u. Physik* (3), vol 6 (1904), p. 104.