THE FEBRUARY MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and forty-second regular meeting of the Society was held in New York City on Saturday, February 27, 1909. A single session sufficed for the brief programme. The attendance included the following twenty-four members:

Professor Maxime Bôcher, Professor E. W. Brown, Professor F. N. Cole, Miss E. B. Cowley, Miss L. E. Cummings, Professor H. B. Fine, Dr. Frank Irwin, Professor Edward Kasner, Mr. W. C. Krathwohl, Mr. H. F. MacNeish, Professor W. F. Osgood, Professor R. W. Prentiss, Mr. H. W. Reddick, Professor L. W. Reid, Mr. L. P. Siceloff, Professor D. E. Smith, Professor P. F. Smith, Dr. Elijah Swift, Professor H. D. Thompson, Dr. M. O. Tripp, Professor Oswald Veblen, Mr. C. B. Walsh, Professor H. S. White, Miss E. C. Williams.

The President of the Society, Professor Maxime Bôcher, occupied the chair. The Council announced the election of the following persons to membership in the Society: Mr. W. T. Campbell, Boston Latin School; Professor W. A. Garrison, Union College; Mr. D. D. Leib, Johns Hopkins University; Professor William Marshall, Purdue University; Mr. J. B. Smith, Richmond, Va., High School; Mr. C. M. Sparrow, Johns Hopkins University. Ten applications for membership in the Society were received.

Professor Bôcher presented his resignation as member of the Editorial Committee of the *Transactions*, to take effect August 15, it being his intention to spend the coming year abroad. Professor Osgood was appointed to fill Professor Bôcher's unexpired term.

The following papers were read at this meeting:

(1) Professor EDWARD KASNER: "Brachistochrones and tautochrones."

(2) Dr. D. C. GILLESPIE: "On extremal curves which are invariant under a continuous point transformation group."

(3) Professor VIRGIL SNYDER: "Infinite discontinuous groups of birational transformations which leave certain surfaces invariant."

Professor Bôcher presented certain results in extension of his paper "On systems of linear differential equations of the first order," read February 22, 1902, and to be published in an early number of the *Transactions*.

In the absence of the authors the papers of Dr. Gillespie and Professor Snyder were read by title. Professor Kasner's paper will appear in the BULLETIN. Abstracts of the other papers follow below, being numbered to correspond to the titles in the list above.

2. Dr. Gillespie studies the connection between an integral, its extremal curves, and a continuous group of point transformations under which they may be invariant.

The condition that a two-parameter family of curves in the plane be the extremal curves of an integral is expressed by a partial differential equation of the first order (a), which the integrand must satisfy.* This system of extremals is the solution of an ordinary differential equation of the second order (b). If the extremals are invariant under a transformation group, there exists at least one first integral of the equation (b) which is invariant under the transformation; *i. e.*, every one parameter family of curves obtained by assigning a value to the arbitrary constant in this first integral is invariant under the transformation.† This first integral together with the transformation defines a solution of the equation (a) which contains an arbitrary function, but which is not necessarily the complete solution of the equation. The relation between this first integral, the transformation, and the system of solutions of (a) can be expressed by an equation. When the first integral is one of the knowns, this is an algebraic equation; when the first integral is the unknown, it is a differential equation, which requires nevertheless only a quadrature for its solution.

3. Only a few isolated examples of surfaces have been found which are invariant under a discontinuous group of birational transformations of infinite order. Professor Snyder shows that these examples all depend upon a certain (2, 2) correspondence. Three interpretations are given. The first applies to quartic surfaces having more than one conical point. In any plane

* Bolza, Lectures on the calculus of variations, p. 31.

† Lie-Scheffers, Vorlesungen über Differentialgleichungen, p. 375. Page, Ordinary differential equations, p. 149.

through two conical points the (2, 2) correspondence is defined by the pencils through the nodes. The second also refers to quartic surfaces, those having two nets of hyperelliptic curves. The (2, 2) correspondence is defined by the lines joining the points of the canonical g_2^1. The third concerns the systems of bitangents on any surface which is complete focal surface of two or more congruences.

F. N. COLE,
Secretary.

BÉZOUT'S THEORY OF RESULTANTS AND ITS INFLUENCE ON GEOMETRY.

PRESIDENTIAL ADDRESS DELIVERED BEFORE THE AMERICAN MATHEMATICAL SOCIETY DECEMBER 30, 1908.

BY PRESIDENT HENRY S. WHITE.

THE accepted truths of today, even the commonplace truths of any science, were the doubtful or the novel theories of yesterday. Some indeed of prime importance were long esteemed of slight importance and almost forgotten. The first effect of reading in the history of science is a naive astonishment at the darkness of past centuries, but the ultimate effect is a fervent admiration for the progress achieved by former generations, for the triumphs of persistence and of genius. The easy credulity with which a young student supposes that of course every algebraic equation must have a root gives place finally to a delight in the slow conquest of the realm of imaginary numbers, and in the youthful genius of a Gauss who could demonstrate this once obscure fundamental proposition.

The first complete proof, by Gauss, that rational algebraic equations have roots either real or imaginary dates back only to 1799. That part of algebra that is concerned with equations is accordingly for the most part modern, recent indeed, as compared for instance with the plane geometry of lines and circles. Before Gauss, it is true, much had been done in actually solving equations of the lower orders and in the theory of symmetric functions of the roots. After him also the question of the arithmetical character of the roots required not only a Galois to penetrate its mystery, but also a Liouville and a Jordan to expound the marvellous theory that Galois had created.