

the occasional introduction of an equation, which makes the theorems much clearer. In this regard, most of the treatment is less formal but more easily understood than the corresponding part of Reye's treatise.

Since the author holds strictly to the traditions of the Steiner school (though never advocated by Steiner), no figures appear in the book. In many cases the explanations are sufficiently clear without this help, but in discussions of pencils of collineations, products of involutions, and similar subjects, the text could be made much briefer by the addition of appropriate figures.

The fifteen pages devoted to the twisted cubic curve contain a large number of results, skilfully arranged. The chapter on trilinearity is much more systematic than the treatment given this important subject in other places. A number of equations are used to establish the existence of neutral pairs, singular elements, and principal points, but much of the discussion is synthetic. The theory of the cubic surface follows easily from the elementary principles of trilinearity.

Nearly a hundred pages are occupied with the so-called problem of projectivity, *i. e.*, given a series of arbitrary points  $A_k$ , and their associates  $B_k$ , for any point  $P$  to find the corresponding point  $Q$  such that the pencil  $QB_k$  is projective with  $PA_k$ . The problem has no meaning if the number of arbitrary points is less than four, and becomes impossible when it is greater than seven. In particular, the Cremona transformation of degree five having six distinct base points results from five points  $A_k$ . The corresponding problem in space includes the discussion of the tetrahedral complex, and some surfaces belonging to it.

Some birational transformations are found in space which are believed to be new.

The later volumes are to contain the theory of collineations in space of two and of three dimensions, including the singular or degraded forms, and the theory of Cremona transformations and multiple correspondences.

VIRGIL SNYDER.

*Beiträge zur Theorie der linearen Transformationen als Einleitung in die algebraische Invariantentheorie.* By W. SCHEIBNER. Leipzig, B. G. Teubner, 1907. 250 pp.

THE contents of most of this book appeared in the *Sächsische Berichte* from 1903 to 1907. The author explicitly declares

that no new results are obtained and no essentially new method developed, but hopes by the repeated and consistent use of one method to awaken further scientific interest. The subject is developed along lines not too difficult for less mature readers, yet the point of view is sufficiently broad to justify the labor and patience involved in working through the maze of long algebraic processes. No use is made of symbolic notation, except that occasionally the author introduces various abbreviations of his own. A more remarkable restriction is that to non-homogeneous forms. This requires frequent paraphrases and the introduction of numerous unsymmetric formulas that are harder to master than the rather simple idea of homogeneous forms.

The ideas of invariant, covariant, source, hessian, jacobian, polarization, and transvectant are clearly treated in the first thirty pages, thus rapidly introducing the reader to the essence of the subject. The application to cubic and quartic forms is similar to that found in the treatise of Burnside and Panton, though more extensive and more systematic. A short chapter discusses the reduction of elliptic differentials to the canonical form; it is more direct than the procedure frequently followed, but presupposes greater maturity on the part of the reader.

Chapter IV, of 42 pages, is devoted to the determination of the system of invariants and covariants of quintic and sextic binary forms. This is the most interesting chapter in the book, although the goal is not always held clearly in view. The various steps are clear and follow the scheme laid down in the preceding chapters quite closely. The proof of the finiteness of the complete system is not completed, except that it is shown that the number of forms for the quintic cannot exceed twenty-three, and these are shown to be distinct by calculating the sources.

At times a paucity of words and a generous amount of algebraic detail lessen the pleasure of reading, but the subject matter makes any other procedure difficult. Unfortunately no geometric interpretation of the various forms is given, and the entire development proceeds strictly along algebraic lines.

The general subject matter of the book is followed by four appendices which are either applications of the methods discussed, or are concerned with linear transformations of non-algebraic functions. The first, of 10 pages, gives the geometric interpretation of the bilinear transformation in the complex

plane. It is a distinct digression from the rest of the book, and is quite elementary. It was introduced to make the later appendices intelligible, but the treatment in these applications is so condensed that it can be of little value to a reader not having much more familiarity with conformal representation than that provided in the first appendix.

The second, of 20 pages, on the Tschirnhausen transformation, includes a detailed treatment of the reduction of the quintic to the Bring-Jerrard normal form; otherwise it is rather similar to the corresponding portion of Weber's Algebra.

The third appendix of 20 pages considers the solution of the icosahedron equation. The first half gives a very rapid survey of the Schwarzian derivative, the hypergeometric series, and the expression of the constants of transformation by means of gamma functions. The subject proper of this appendix is the working out of the problem suggested in Klein's *Ikosaeder*, page 139, *i. e.*, to start with the general binary quintic and deductively obtain the solution in terms of modular functions. The discussion is followed by a numerical illustration which greatly adds to its clearness.

The last appendix, of 40 pages, is concerned with linear transformations of elliptic theta functions and modular functions. As in the two preceding appendices, the amount of presupposed knowledge is much greater than in the book proper. The treatment is entirely transcendental, and has nothing in common with the preceding portions of the work. Numerous references show the relations between the present development and the existing literature, but it is not clear why this subject should be treated in a book on algebraic invariants.

VIRGIL SNYDER.

*Table de Caractéristiques relatives a la Base 2310 des Facteurs premiers d'un Nombre inférieure a 30,030.* By ERNEST LEBON. Paris, Delalain Frères, 1906. 32 pp.

IN this pamphlet the author has published a table of "characteristics with respect to the base 2310" by means of which any number between 1 and 30,030 can be readily factored. The first twelve pages are devoted to an explanation of the simple theory upon which the usefulness of the table is based, and to a description of the devices by means of which the characteristics are calculated. The last twenty pages contain the table itself. In a recent number of the BULLETIN, Professor