SHORTER NOTICES.

Encyklopädie der Elementar-Mathematik. Ein Handbuch für Lehrer und Studierende. Von HEINRICH WEBER und JOSEF WELLSTEIN. Zweiter Band, *Elementare Geometrie.* Bearbeitet von HEINRICH WEBER, JOSEF WELLSTEIN, und WALTHER JACOBSTHAL. Leipzig, Teubner, 1905. 8vo. 280 figures. viii + 602 pp. Dritter Band, *Angewandte Elementar-Mathematik.* Bearbeitet von HEINRICH WEBER, JOSEF WELLSTEIN, und RUDOLF H. WEBER. Leipzig, Teubner, 1907. 8vo. 358 figures. ix + 666 pp.

EVERY live teacher of secondary school mathematics is aware of the superficial character of most text-books. In the nature of the case, teaching for younger pupils must exclude far more than it presents. But if the teacher himself restricts his study to the range prescribed for pupils, very little mathematical interest is kindled in his classes. Practically the same result is reached if the sole scientific interest of the teacher is in fields remote from his pupils' studies. The authors of this three-volume encyclopedia of elementary mathematics plan to intensify by fundamental criticism, and revivify by extensive applications to questions of physics, the interest of the young teacher in his every-day work. This does not conflict by any means with the programme of modern universities — to train the future teacher by research in some region on the frontiers of scientific knowledge. Rather it supplements that programme, and strengthens the position of its champions, by showing how to apply the method of the university seminar to the problems of the schoolroom.

The book on geometry begins with a critical and historical survey of the notions point, straight line, surface, plane, parallel. The antithesis is developed between what the authors call natural and logical geometry, the latter reached only by a limit process of idealization. Most appropriate is then the quite full examination of a second system of geometrical objects, the totality of spherical surfaces that contain a fixed point O; for in this system there is an exact correspondence to the objects of ordinary euclidean geometry, while the images are radically different. Straight lines are replaced by circles through O, and planes by spheres through O. Tried upon this material, the Hilbert axioms of connection and of order lose their appearance of artificiality and become novel observations of fact. Then an exchange (duality) of sphere for point, sheaf of spheres for line, etc., serves to shake loose the habitual prepossession that point must mean always one thing, and that the truth of the theorems of Euclid's geometry is mystically dependent upon something more than the axioms. Soon is introduced also the three-fold infinity of circles in a plane: each circle is called a point, each pencil of circles a line, and each "Bündel" a plane. It is easy thence to foresee how there must arise elliptic and hyperbolic non-euclidean geometries in a plane or on a spherical surface.

By easy stages the reader is led to Hilbert's inquiries respecting the independence of axioms, and to the "pathological" systems constructed for the purpose of establishing that independence. The climax of this fascinating chapter is reached in the paragraph 14, where metageometry is considered in its relation to philosophy, especially to Kant. It will interest the casual reader to note (page 144) the explicit opinion: "Necessities of intuition there are none; necessity can lie only in acts of the intellect;" while a footnote states that on this point there is a difference of opinion between the two editors.

Projective geometry is treated no less carefully as to its axioms, conics are considered in their main features — and shown to be central projections of circles, and projective metric concludes this third section, to which is added a well chosen list of references.

Planimetry (pages 220–301), shows the same influence of Hilbert's "Foundations." In particular it contains a section that will be welcome to very many teachers of geometry, upon the number π and the history of circle measurement. Inversion with respect to a circle, and the problem of Apollonius appear as extensions of the traditional school-book material.

Trigonometry, plane and spherical, is given a concise discussion. New to many will be the division of formulas into those of the first order and those of the second, and the generalization of the spherical triangle (Gauss and Study), with Study's theorem that the totalities of proper and of improper spherical triangles form two discrete continua, with no continuous transition from the one to the other by moving the vertices freely in the surface of the sphere. Three points of the surface denote a great number of different triangles when sides and angles are unrestricted in magnitude, and for both the distinction is maintained between positive and negative. Adopting any modulus, as 2π, 4π, 6π, etc., renders the number of non-equivalent classes of triangles with three fixed vertices finite, and Study's theorem is valid for all moduli.

Brief but artistic sections on analytic geometry of the plane (76 pages) and of space (72 pages) include much of value, as the elements of integration for volume, and the rotation groups of regular solids. A good index, and a full supply of clear diagrams, make this a valuable book of reference even for teachers who will read it but infrequently.

Of the third volume it is not too much to say that it contains a most valuable presentation of physical theories for mathematical teaching. That it is kept free from overloading of theory is seen perhaps in the fact that continuity and discontinuity are not mentioned in the index, nor critical states of matter. Half the volume is physics, vector geometry, analytical statics, dynamics, electricity and magnetism, and electromagnetism. Of the remainder, maxima and minima in geometry and capillarity fill 43 pages; probability and least squares, 40 pages; and a full and suggestive book on graphical statics the concluding 240 pages.

In a note appended to this volume, H. Weber reverts to the Mengenlehre of the first volume, cites Russell's paradox on the class of classes that do not contain themselves (which he identifies with one of Kant's antinomies); and gives an outline discussion of finite aggregates, free from objections, as he believes. These volumes certainly constitute a valuable work for every reference library. H. S. WHITE.

Leçons sur l'Intégration et la Recherche des Fonctions Primitives. Par HENRI LEBESGUE. Paris, Gauthier-Villars, 1904. 8vo. viii + 138 pp.

SINCE the publication of Lebesgue's thesis in 1902 the originality and power of his methods have attracted increasing attention to the field in which he and Baire have made such important contributions. They have given to the study of discontinuous functions an impulse which is apparent on the most cursory survey of current mathematical periodicals, and of such recent treatises as those of Young and Hobson.

The present volume, one of the series of monographs published under the direction of Borel, reproduces a course of twenty lectures delivered at the Collège de France on the Peccot foundation. Within such limits one could hardly ex-