

THE APRIL MEETING OF THE AMERICAN
MATHEMATICAL SOCIETY.

THE one hundred and thirty-eighth regular meeting of the Society was held in New York City on Saturday, April 25, 1908, extending through the usual morning and afternoon sessions. The attendance included the following thirty-five members :

Professor G. A. Bliss, Professor Maxime Bôcher, Professor F. N. Cole, Professor L. P. Eisenhart, Professor G. H. Hallett, Mr. G. W. Hartwell, Dr. A. A. Himowich, Professor E. V. Huntington, Professor J. I. Hutchinson, Professor Edward Kasner, Professor C. J. Keyser, Mr. E. H. Koch, Jr., Mr. W. C. Krathwohl, Professor P. A. Lambert, Dr. G. H. Ling, Mr. L. L. Locke, Mr. E. B. Lytle, Dr. Emory McClintock, Professor Max Mason, Mr. A. R. Maxson, Dr. R. L. Moore, Professor Richard Morris, Professor W. F. Osgood, Mr. H. W. Reddick, Mr. E. W. Sheldon, Mr. L. P. Sicheloff, Professor S. E. Slocum, Mr. F. H. Smith, Professor P. F. Smith, Dr. C. E. Stromquist, Professor H. D. Thompson, Mr. C. A. Toussaint, Professor Oswald Veblen, Professor H. S. White, Professor J. W. Young.

The President of the Society, Professor H. S. White, occupied the chair, being relieved at the afternoon session by Professor C. J. Keyser. The Council announced the election of the following persons to membership in the Society : Professor H. E. Buchanan, Lincoln College, Lincoln, Ill.; Mr. E. F. A. Carey, University of California ; Professor F. E. Chapman, Southern University, Greensboro, Ala.; Professor R. C. Maclaurin, Columbia University ; Mr. E. J. Miles, University of Chicago ; Mr. C. A. Stiles, University Preparatory School, Ithaca, N. Y.; Mr. J. S. Thompson, Mutual Life Insurance Company, New York, N. Y.; Mr. O. A. Turney, Phoenix, Ariz.; Mr. C. B. Walsh, Ethical Culture School, New York, N. Y.; Professor R. T. Wilbur, Christian Brothers College, St. Louis, Mo.; Miss E. R. Worthington, Yale University. Ten applications for admission to membership in the Society were received.

At the meeting of the Council Professor E. B. Van Vleck was reelected a member of the Editorial Committee of the *Transactions*, to serve for the three years 1908-1911. It was

decided to hold the summer meeting and colloquium of the Society in 1909 at Princeton University, and Professors Fine, Osgood, Holgate, and the Secretary were appointed a committee to make the appropriate arrangements. A committee consisting of Professor P. F. Smith, C. J. Keyser, and G. A. Bliss was appointed to consider the question of holding the next annual meeting of the Society at Baltimore in affiliation with the American association for the advancement of science.

The following papers were read at this meeting:

(1) Professor S. E. SLOCUM: "The collapse of tubes under external pressure."

(2) Professor E. B. WILSON: "On the differential equations of the equilibrium of an inextensible string."

(3) Professor E. B. WILSON: "On the principle of relativity."

(4) DR. ELIJAH SWIFT: "Note on the second variation in an isoperimetric problem."

(5) Professor J. I. HUTCHINSON: "The hypergeometric functions of n variables."

(6) Professor EDWARD KASNER: "Note on Meusnier's theorem."

(7) Professor B. F. FINKEL: "Determination of the groups of order 2^m which contain selfconjugate cyclic subgroups of order 2^{m-4} and whose generating operations correspond to the partitions $[m-4, 4]$, $[m-4, 3, 1]$."

(8) Professor J. W. YOUNG: "Two-dimensional chains and the classification of complex collineations in a plane."

(9) Mr. C. N. MOORE: "On certain constants analogous to Fourier's constants."

(10) Professor PAUL SAUREL: "On the distance from a point to a surface."

(11) Mr. E. B. LYTLE: "Multiple integrals over iterable fields."

(12) Professor P. A. LAMBERT: "The fundamental theorem of algebra."

(13) Professor E. V. HUNTINGTON and Mr. H. P. FORTÉ: "On the fluctuations in the speed of a flywheel."

(14) Professor E. V. HUNTINGTON: "On the theory of the gyroscope, with special reference to the Brennan monorail car" (preliminary communication).

(15) Dr. O. E. GLENN: "Studies in the theory of degenerate algebraic curves" (preliminary communication).

In the absence of the authors the papers of Professor Wilson, Dr. Swift, Professor Finkel, Mr. Moore, Professor Saurel, and Dr. Glenn were read by title. The papers of Dr. Swift and Mr. Moore appeared in the May BULLETIN. Professor Saurel's paper will be published in the July number of the BULLETIN. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Professor Slocum's paper is an analysis of experimental data on the collapse of tubes under external pressure. The material for this analysis is found in the recent extensive experiments made independently by Professor Carman and Professor Stewart.

The rational formulas for thin and thick tubes are discussed, and the significance of the elastic constants involved, as indicating the nature of collapse, is pointed out. The fact that the physical properties of the material, given in connection with the experiments in question, did not include these elastic constants, indicates the irrational nature of the results and their consequent limitations. The characteristic difference between the failure of thin and thick tubes is also explained.

The nature and amount of the deviation of commercial tubes from the ideal assumptions upon which the rational formulas are based is examined and corrections determined. In this connection the flexibility of rational formulas as applying to particular cases, and the rigidity of empirical formulas as merely striking an average, are contrasted in terms of numerical percentages.

A graphical comparison of the curves represented by the rational formulas for thin and thick tubes is made for steel and for brass. The limit for use is determined as the abscissa of the point of intersection of these curves, and is found to agree closely with the empirical values determined experimentally.

A graphical comparison is also made between Stewart's empirical formulas and the corrected rational formulas for thin and thick tubes, the former representing an ellipse and its linear tangent, and the latter a cubic curve and a parabola respectively.

The results of the analysis are embodied in the formulas

$$P = C \frac{2E}{1 - m^2} \left(\frac{t}{D} \right)^3 \text{ for thin tubes,}$$

$$P = 2u_c K \frac{t}{D} \left(1 - K \frac{t}{D} \right) \text{ for thick tubes,}$$

where E = Young's modulus, m = Poisson's ratio, u_c = ultimate compressive strength, P = external collapsing pressure, t = thickness, D = diameter, and C and K = correction constants, all dimensions being expressed in inches and pounds as units. C and K are so expressed that they may be determined independently for individual specimens, or for the various types of commercial tubes. From the data under discussion C and K are found to have the following values: For lap welded steel boiler flues, $C = .69$ and $K = .89$; for cold drawn seamless steel flues, $C = .76$; for drawn seamless brass tubes, $C = .78$.

The article will appear in full in the *Engineering News*.

2. Professor Wilson treats the differential equations of the equilibrium of an inextensible string from the point of view of Lie's theory of groups. He obtains the conditions that the equations should admit a group of eight parameters, and thereby determines a case in which the equations are completely integrable by quadratures and where the field of force is neither central nor parallel. The paper will be offered to the *Transactions*.

3. In a short note, Professor Wilson discusses the principle of relativity in electrodynamics with reference to recent suggestions and recent experimental developments. The paper will be offered to the *Philosophical Magazine*.

5. The $n + 1$ linearly independent hypergeometric integrals regarded as functions of n parameters x_i entering into the integrand are considered in the paper by Professor Hutchinson. The chief problem solved is the determination of the group of homogeneous linear transformations which these integrals undergo when the x_i describe closed paths. It is also shown that a certain hermitian form is invariant for this group.

6. Professor Kasner shows that the most general system of space curves for which Meusnier's theorem is valid is defined by a differential equation of the type $\alpha y'' + \beta z'' + \gamma = 0$, where the coefficients are arbitrary functions of x, y, z, y', z' . This includes Lie's extension as a special case. Such systems arise in connection with the general transformation of lineal elements.

8. Fundamental in the classification of the linear fractional transformations on one complex variable — *i. e.*, of the projectivities on a complex line — is the notion of a chain of points on the line. A chain may be defined as any class of points projective with the real points of the line. If the complex numbers be represented in the usual manner by the real points of a plane, the chains of the complex line are represented by the circles of this plane. The behavior of the projectivities on the line with reference to the chains of the line then leads readily to the well-known classification of these projectivities into hyperbolic, elliptic, etc.*

The corresponding classification of the projective transformations in a complex space of n dimensions must lead first to a generalization of the notion of a chain. It is obtained naturally as follows: Using homogeneous coordinates, any point in a complex space S_n of n dimensions is represented by $(x_0, x_1, x_2, \dots, x_n)$, where the x_i are any complex numbers (the combination $(0, 0, \dots, 0)$ of course excluded). The totality of such points in which the x_i are any real numbers forms the real subspace R_n of n dimensions in S_n . Any subspace of S_n obtained by subjecting R_n to any linear homogeneous transformation on the x_i with complex coefficients Professor Young calls an n -dimensional chain in S_n , or more briefly an n -chain.† In the present paper Professor Young considers in detail the case $n = 2$. Certain fundamental properties of 2-chains in a complex plane are first developed, which are then applied to effect the desired classification “with respect to reality” of the complex collineations in a plane. Each of the well-known five types of collineations in the plane is subdivided with reference to the existence of invariant 2-chains, and the existence of each subtype is established. This classification is of interest not merely in the field of projective geometry and continuous groups, but also in the theory of automorphic functions of two variables. The treatment is largely synthetic and the proofs are so simple as to make the methods applicable to larger values of n with comparatively little increase in complexity.

11. Former discussions of the problem of the reduction of

* For a synthetic treatment of the one-dimensional case, cf. a paper by the author presented to the Society at its last (February) meeting on “The geometry of chains on a complex line.”

† For a non-analytic definition of this notion, cf. “A system of assumptions for projective geometry,” by O. Veblen and J. W. Young, presented to the Society at its last Christmas meeting.

multiple integrals to iterated integrals have been restricted to functions defined over Jordan measurable fields. Mr. Lytle defines iterable fields in such a way as to include all measurable and a large class of non-measurable aggregates. By means of this new class of iterable aggregates important relations between multiple and iterated integrals are derived. These relations lead to a very general condition for inversion of iterated integrals.

12. The purpose of Professor Lambert's paper is to present a general method for determining all the roots of any algebraic equation by means of infinite series. The method consists in forming three algebraic functions of x from the given equation (1) $f(y) = 0$, (a) by introducing a factor x into all the terms of (1) except the first and last; (b) by introducing a factor x into all the terms of (1) except the first and second; (c) by introducing a factor x into all the terms of (1) except the second and last.

These algebraic functions are expanded into power series in x by Laplace's series. If in these power series x is made unity, the resulting series, if convergent, determine the roots of the given equation. It is shown that all the roots of the algebraic equation can be expressed in infinite series derived either from the algebraic function formed in accordance with (a), or from the two algebraic functions formed in accordance with (b) and (c).

13. The paper of Professor Huntington and Mr. Forté contains an exact formula for the small fluctuations in the speed of the fly wheel of a horizontal engine, with the results of illustrative experiments made in the Harvard engineering laboratory by the use of a tachograph.

Let N be the number of revolutions per minute, regarded as a function of the crank angle θ , and suppose that $N = N_1$ when $\theta = \theta_1$. Then the value of N for any other value of θ is given by

$$N^2 = N_1^2 \left\{ 1 + \frac{1800g \left(\frac{U}{W_p r} \right) \Big|_{\theta=\theta_1}^{\theta=\theta} - \left(A^2 + \frac{w}{W_p} B^2 \right) \Big|_{\theta=\theta_1}^{\theta=\theta}}{\frac{W}{W_p} \frac{K^2}{r^2} + A^2 + \frac{w}{W_p} B^2} \right\},$$

where A and B are numerical functions of θ depending on the

proportions of the engine, and U is the total work done by the external forces (steam pressure, load, gravity, and friction) during the interval from $\theta = 0$ to $\theta = \theta$. Here W , W_p , w are the weights of flywheel, piston, and connecting rod, K = radius of gyration of the flywheel, and r = crank radius. The values of A and B are as follows :

$$A = \frac{\sin(\theta + \phi)}{\cos \phi}, \text{ where } \phi \text{ is given by } \sin \phi = \frac{r}{l} \sin \theta;$$

$$B^2 = \sin^2 \theta \left[1 + \frac{l - c}{l} \frac{r \cos \theta}{\cos \phi} \right]^2 + \frac{c^2}{l^2} \cos^2 \theta + \frac{k^2}{r^2} \left(\frac{r \cos \theta}{\cos \phi} \right)^2.$$

Here l = length of connecting rod, c = distance from wrist pin to center of gravity, and k = radius of gyration about the center of gravity. For the value of U , if the effective pressure P on the piston and the resisting moment RL of the load L about the shaft are known functions of θ , and if the expression for the work of friction is assumed to be of the form $(F + fL)R$, where F and f are constants, then

$$U = \int_0^\theta [PdX - RLd\theta - (F + fL)Rd\theta] - wc \sin \phi,$$

where $X = r(1 - \cos \theta) + l(1 - \cos \phi)$, so that $dX = rAd\theta$.

The formula for N holds good for all conditions of pressure and load; the condition for "uniform speed" is that the integral in the expression for U shall reduce to zero at the end of each revolution.

14. Professor Huntington's second paper, when completed, will contain approximate solutions of the equations of motion which present themselves in the theory of the gyrostatic balancing of the Brennan monorail car.

15. The general subject of the relations which exist between an algebraic curve and those curves of lower degree into which it can decompose has been studied only incidentally, or at best only in connection with special curves of low degrees. Certain results have been published by Cayley, Zeuthen, Taylor, and others; but their work is only fragmentary. Dr. Glenn has organized a general theory of the decomposition of curves of n th degree, and presents, in the form of a preliminary report approximately half of the results of his investigation.

The report contains chapters on the following topics: the asymptotic properties of ternary homogenous forms; the penultimate forms of a curve; the existence of "free summits" on a degenerate algebraic curve; the classification of curves on the basis of the penultimate forms. Further investigations are in progress. The memoir, as a whole, will be offered for publication the coming year.

F. N. COLE,
Secretary.

THE APRIL MEETING OF THE CHICAGO SECTION.

THE twenty-third regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of Chicago, on Friday and Saturday, April 17-18, 1908. One session was held on Friday and two on Saturday. On Friday evening twenty of the members dined together informally in the private dining room of the University of Chicago Commons. The attendance at the three sessions included forty-five persons, among whom were the following thirty-three members:

Dr. G. D. Birkhoff, Professor Oscar Bolza, Dr. R. L. Börger, Professor H. E. Buchanan, Mr. Thomas Buck, Professor W. H. Bussey, Dr. A. R. Crathorne, Professor D. R. Curtiss, Professor L. E. Dickson, Mr. Arnold Dresden, Mr. E. P. R. Duval, Professor W. B. Ford, Mr. R. M. Ginnings, Professor Harriet E. Glazier, Professor C. N. Haskins, Mr. T. H. Hildebrandt, Mr. F. H. Hodge, Dr. A. C. Lunn, Mr. W. D. MacMillan, Professor G. A. Miller, Dr. J. C. Morehead, Professor F. R. Moulton, Dr. L. I. Neikirk, Professor H. L. Rietz, Miss Ida M. Schottenfels, Mr. A. R. Schweitzer, Professor H. E. Slaught, Dr. Clara E. Smith, Dr. A. L. Underhill, Professor E. B. Van Vleck, Dr. A. E. Young, Professor J. W. A. Young, Professor Alexander Ziwet.

Professor G. A. Miller, Vice-President of the Society and chairman of the Section, presided at all of the sessions. In opening the meeting he spoke of the great loss sustained by the Society in the recent death of Professor Heinrich Maschke. A committee, consisting of Professors E. B. Van Vleck, Alexander Ziwet, and H. E. Slaught, was appointed to draft suitable resolutions on behalf of the Section, and these were pre-