scheme may be regarded as analogous to the modern method of infinitesimals when founded upon the doctrine of limits. There is a normal and systematic procedure, although tedious and laborious, for converting the mechanical proofs into exhaustion proofs. Therefore Archimedes may be regarded as having taken the decisive step in founding a method which in essential respects is that of the integral calculus. If he had been like many modern mathematicians, he would have omitted the exhaustion proofs altogether, but would have added to each of his mechanical proofs a set phrase like this: "It is easy to see that an exhaustion proof may be constructed in the usual manner. This is left as an exercise for the reader."

CHARLES S. SLICHTER.

SHORTER NOTICES.

Mehrdimensionale Geometrie, II Teil, *Die Polytope*. Von PROFESSOR DR. P. H. SCHOUTE. Leipzig, G. J. Göschen (Sammlung Schubert XXXVI). 1905. ix + 326 pp.

THE second (and final) volume of this work, like the first, is worthy to be associated with the other excellent books of the Schubert collection. Comparatively little of the subject matter is new, but a large number of interesting and useful results have been gathered together in a convenient form. The entire volume is devoted to the treatment of the polytop, which the author defines, for space of n dimensions, as any portion of that space enclosed in any manner whatever. The first 262 pages treat the linear polytop, *i. e.*, one bounded by flat spreads of $n - 1$ dimensions (R_{n-1}'s); while the remaining 64 pages are concerned with the hypersphere, cone, cylinder, and rotation spread.

Under the heading "Topologische Einleitung," the first section treats (among other things) the simplex, and its various sections and projections; the question of a general classification of polytops; the definition of hyper-pyramids and prisms and the n-dimensional analogues of other special polyhedra, such as the truncated pyramid and prism, the frustums, etc.; and finally discusses the Euler law and its n-dimensional extension. In this treatment of the Euler law, thirteen pages are devoted to the well-known three-dimensional case, four different methods of proof being given, while the extension to n dimensions is disposed of in five pages. In view of the difficulty which the author mentions in the preface of reducing the volume to the required number of pages, it would seem that some space might have been saved to advantage at this point.

The second section is devoted to "Massverhältnisse." Congruence, both positive and negative, and similarity are defined, first for the simplex and then for the general linear polytop. Attention is called to three points metrically related to the simplex, viz., the in-center, the circumcenter, and the centroid or center of gravity. One regrets here that the author was forced to omit his originally intended paragraph on the geometry of the simplex. The orthocenter, so closely related to the three points mentioned, might well have been included. The important subject of the section is that of the content (Inhalt) of polytops. Formulas are deduced for the content of various types of polytops in the following order: The rectangular parallelotop, the general parallelotop, the prism, the simple pyramid, the simplex, etc., the general procedure being similar to the treatment in our elementary geometries. A convenient expression in determinant form for the content of a simplex in terms of the lengths of its $\binom{n+1}{2}$ edges is deduced.

"Reguläre Polytope" is the subject of the third section; and here again, in a work on higher-dimensional geometry, much less attention might have been given to the two- and three-dimensional cases. The larger part of the fifteen and forty-five pages devoted respectively to the regular polygon and regulae polyhedron might have been more advantageously used for some of the material which the author was forced to omit in condensing his manuscript. The treatment of the higher-dimensional polytops furnishes, in the estimation of the writer, the two most interesting chapters of the book. In one chapter the six four-dimensional figures are considered, the simplest sets of rectangular cartesian coordinates of the vertices are given for each, the Schlegel diagrams and various three-dimensional sections are studied, and the group connected with each figure is given. In the next chapter the three regular polytops which exist in space of any number of dimensions are studied. In a convenient little table one finds, for each of the three figures, formulas for the radii of the n associated hyper-spheres, the angle between two adjacent R_{n-1}'s, the content, and the boundary content (Oberfläche), all in terms of the length of an edge. The general expression for the order of the group associated with each of these figures is also given. The omission of a phrase from the theorem at the top of page 256 is easily corrected by reference to the formula at the bottom of the preceding page.

The fourth and final section treats "Die runden Polytope." In the work on the hyper-sphere one notices such topics as, the sphere passing through $n + 1$ points, the sphere touching $n + 1$ R_{n-1}'s, the sphere touching $n + 1$ other spheres, the configuration of the centers of similitude of $n + 1$ spheres, the content and surface content of the sphere, and the content of the spherical sector and segment. The hyper-cone and cylinder are similarly treated. Under general rotation figures, one finds the quadric spreads generated by revolving a flat spread about a flat spread as an axis, the torus spreads and the Guldin spreads obtained by revolving hyperspheres and linear polytops respectively.

Since the book is largely a compilation of previously known results, one regrets that references to the literature of the subject are not more numerous and specific. This second volume is better than the first in this respect, but still leaves much to be desired.

The book will doubtless prove to be a valuable reference work to those who are interested in, and have use for, the metrical formulas of higher-dimensional geometry; but many readers will doubtless share with the present writer a regret that the author's point of view has been so largely metrical, both in his choice of topics and in his method of treatment.

W. B. CARVER.

Theory of the Algebraic Functions of a Complex Variable. By J. C. FIELDS. Berlin, Mayer & Müller, 1906. v + 186 pp.

THE work before us is not intended as a treatise or text-book on the theory of algebraic functions along any of the well-established lines of treatment. It is, on the contrary, a new and distinctive mode of approach to this class of functions, although grounded on principles which in their essence are already familiar. The methods employed are purely algebraic, we might almost say arithmetic, in character, and in this respect the influence of Weierstrass may be said to predominate.

The fundamental idea on which the work is based is the notion of "order of coincidence." A given class of algebraic functions is defined as usual by a rational expression in (z, v)