THE FEBRUARY MEETING OF THE SAN FRANCISCO SECTION.

THE thirteenth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held on Saturday, February 29, 1908, at Stanford University. Professor Hoskins presided. The following thirteen members were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor R. L. Green, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Mr. Joseph Lipke, Dr. J. H. McDonald, Professor W. A. Manning, Professor Irving Stringham, Mr. W. H. Stager, Mr. J. D. Suter, Professor S. D. Townley.

The following papers were read at this meeting:

(1) Professor D. N. LEHMER: "A discussion by synthetic methods of the covariant conic of two given conics."

(2) Mr. JOSEPH LIPKE: "Ein Fehler im König'schen Beweise des Reciprocitätsgesetzes in der Theorie der quadratischen Reste."

(3) Professor G. A. MILLER: "Generalization of the positive and negative numbers." (By title.)

(4) Professor L. M. HOSKINS: "A general diagrammatic method of representing propositions and inference in the logic of classes."

(5) Dr. J. H. MCDONALD: "On certain types of continued fractions considered from a common point of view."

(6) Mr. J. D. SUTER: "On the surface $F_1(u) = u/(1 - u^4)$, $F_2(v) = v/(1 - v^4)$" (preliminary report).

(7) Professor H. F. BLICHFELDT: "On a certain basis of geometry" (preliminary report).

(8) Professor L. M. HOSKINS: "General algebraic solutions in the logic of classes."

Abstracts of the papers follow below in the order as given above.

1. Professor Lehmer gave a discussion by synthetic methods of the problem: To find the locus of a point in the plane such that the four tangents from it to two fixed conics shall be harmonic. The proof is made to depend on the theorem: Given a point row of the second order and a pencil of rays of the second order projective to it, at most four rays of the pencil

will pass through the points on the point row which correspond to them.

2. In the *Acta Mathematica*, volume 22, pages 181–191, Professor König gives a proof of the theorem of reciprocity in the theory of quadratic residues, a proof by complete induction but without the use of the famous gaussian lemma, according to which, if p is a prime of the form $8n + 1$, there always exists a prime q less than $2\sqrt{p} + 1$ such that p is a quadratic non-residue of q. In this note, Mr. Lipke points out an error in Professor König's work which invalidates the latter's proof of the theorem, and also illustrates by an example that there is something radically wrong with the method of proof adopted.

3. Calling the totality of the ordinary complex numbers which have α_0 for their amplitude the α_0-numbers, or the α_0 ray of numbers, Professor Miller considers the various sets of rays in regard to group properties with respect to the fundamental operations of arithmetic. In accord with this general nomenclature, the positive numbers are called 0-numbers and the negative ones are called π-numbers, the number 0 being a special 0-number. Each of two rays whose amplitudes differ by π is said to be the extension of the other, and it is observed that a ray and its extension always form a group with respect to addition or subtraction, but that no other finite number of rays has this property. The common rule for adding a positive and a negative number is included in the following: To find the sum of a number on a ray and a number on its extension, take the difference of their absolute values and prefix the angle of the one which has the larger absolute value. If a set of numbers forms a group with respect to multiplication or division, it involves either only one number from each ray represented in the set or it involves an infinite number of numbers from each one of these rays; and if a finite number of rays forms a group with respect to these operations, it includes the ray of 0-numbers. The 0-numbers and the π-numbers constitute the only finite number of rays forming a group with respect to both of the operations addition and multiplication, and this is one reason why these numbers are of such special importance. If a finite number of rays forms a group with respect to multiplication or division, these rays as units form a cyclic group. The paper will appear in the *American Mathematical Monthly*.

4. The character of the general solutions referred to by Professor Hoskins is illustrated by what may be called the generalized syllogism, which may be stated as follows: Given any two propositions involving x, y and y, z respectively, it is required to infer a proposition involving x, z. The three propositions being written in the most general equational form, according to Boole's symbolism, the result of the solution is so to express the coefficients that when the premises are given it is seen at a glance which (if any) of the coefficients in the conclusion are determined. The method of solution consists in regarding each of the three propositions forming the premises and the conclusion as derived by elimination from the general proposition involving x, y, z. The general result includes both universal and particular (or existential) propositions.

5. This paper defines by means of certain inequalities the manner in which the development in a continued fraction may be formed. These inequalities lead to three distinct types. Dr. McDonald has investigated the rapidity of convergence of each type.

6. Mr. Suter discussed certain properties of the surface $F_1(u) = u/(1 - u^4)$, $F_2(v) = v/(1 - v^4)$ by means of a system of equations of the form

$$(a)\ x = \phi_1(\xi, \eta), \quad (b)\ y = \phi_2(\xi, \eta), \quad (c)\ z = \phi_3(\xi, \eta),$$

found by substituting the given functions in Weierstrass's formulas for expressing the rectangular coordinates of any point on a minimal surface in terms of the imaginary parameters $u = \xi + i\eta$, $v = \xi - i\eta$. He showed that every pair of values (ξ, η) which satisfies the hyperbola $\eta^2 - 2\xi\eta\alpha^{-1} - \xi^2 + 1 = 0$, gives by substitution in (a) and (c), points on the section of the surface with the plane $y = -\frac{1}{2}\tan^{-1}\alpha$; and suggested in this manner a method for modelling the surface; namely, by constructing sections in planes parallel to the xz plane.

7. Only a finite number of types of trigonometries are possible under the following set of assumptions made in regard to a system of an infinite number of points (which we call a plane):

(1) With every pair of points a, b of the system belong three numbers; one we call the distance, (ab) or (ba); and the other two directions, $\overline{ab}$ and $\overline{ba}$ respectively.

(2) The existence of a trigonometry is assumed. If a, b, c are any three points of the system, we postulate a relation of the form

$$\overline{ba} - \overline{bc} = f[(ab), (bc), (ac)],$$

where $f(\alpha, \beta, \gamma)$ satisfies certain conditions as to continuity, etc., when α, β, γ are considered as independent variables.

(3) If c is a constant, and α_1, α_2, β_1, β_2 independent variables, the equations

$$f(\alpha_1, \beta_1, \gamma) + f(\beta_1, c, \beta_2) + f(c, \alpha_1, \alpha_2) = 0,$$
$$f(\alpha_2, \beta_2, \gamma) + f(\beta_2, c, \beta_1) + f(c, \alpha_2, \alpha_1) = 0,$$

can be solved for γ and the solutions are identical.

Professor Blichfeldt remarked that the conditions (2) and (3) have evident geometric meanings in the ordinary euclidean and non-euclidean planes.

8. The representation proposed by Professor Hoskins depends upon an analogy which may be explained by reference to the case of three class terms x, y, z. Let the three pairs of opposite faces of a cube be regarded as representing (x, x'), (y, y'), (z, z') respectively (x' being written for not-x, etc.); then the domain common to any two of these six classes may be represented by the edge determined by the intersection of their representative planes, while the point of intersection of any three of the six planes may represent one of the eight subclasses xyz, xyz', $\cdots$. If each face of the cube is marked with the corresponding class symbol, the relations among the subclasses are seen at a glance. A convenient plane diagram is obtained by drawing a projection of a cube, with a circular space at each vertex which in any particular case can be marked in accordance with the import of a given proposition. A symmetric representation for the case of n primary class terms would require space of n dimensions. But diagrams for practical use may be obtained by repeating the diagram for three terms. Thus the case of six terms would be represented by eight similar cubes, each at a vertex of a larger cube.

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Secretary of the Section.