THE FIFTY-EIGHTH MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE fifty-eighth meeting of the American Association for the Advancement of Science was held at the University of Chicago during the convocation week December 30, 1907, to January 4, 1908. The president of the meeting was Professor E. L Nichols, Cornell University. The address of the retiring president, Dr. W. H. Welch, entitled "The interdependence of medicine and other sciences of nature," was given in the Leon Mandel Assembly Hall on the evening of the opening day.

The meetings of Section A were of unusual interest in view of enthusiastic joint sessions with Section D and the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY to consider the present status and means of improvement of the teaching of engineering students of elementary mathematics, and also to listen to the address of the retiring vice-president, Professor Edward Kasner, of Columbia University, entitled "Geometry and mechanics." The officers of the section were: vice-president, E. O. Lovett; secretary, G. A. Miller; councilor, G. B. Halsted; member of the general committee, F. R. Moulton; sectional committee, Edward Kasner, E. B. Frost, E. O. Lovett, Harris Hancock, F. R. Moulton, E. W. Brown, and G. A. Miller. The following program was presented at the separate meetings of Section A:

(1) Professor E. B. FROST: "Observations with the Bruce spectrograph."

(2) Professor E. B. FROST: "Comments on the Zeiss stereocomparator and the spectrocomparator belonging to the Yerkes observatory."

(3) Professor E. E. BARNARD: "Photographic phenomena of comet *d*, 1907 (Daniel)."

(4) Professor E. E. BARNARD: "On a great bed of nebulosity in Sagittarius, photographed with the Bruce telescope of the Yerkes observatory."

(5) Professor E. O. LOVETT: "Note on the problem of three bodies."

(6) Professor JOEL STEBBINS: "The light curve of Delta Cephei."

(7) Professor JOEL STEBBINS and Mr. F. C. BROWN: "An application of the selenium cell to astronomical photometry."

(8) Professor E. E. BARNARD: "Observations and explanation of the phenomena seen at the disappearances of the rings of Saturn."

(9) Professor E. B. FROST and Mr. PHILIP FOX: "On the twenty-foot horizontal solar spectrograph of the Yerkes observatory."

(10) Professor KURT LAVES: "A graphic method for the determination of the orbit of a spectroscopic binary."

(11) Professor KURT LAVES: "New tables for the time of sight correction of the earth's orbital motion."

(12) Professor W. D. CAIRNS: "A generalized theory of integral equations."

(13) Professor G. B. HALSTED: "On the theory of order, static and nascent."

(14) Dr. F. W. REED: "Singular points in the approximate development of the perturbative function."

(15) Professor JOHN EIESLAND: "On a certain class of algebraic translation surfaces."

(16) Dr. ARTHUR RANUM: "Matrices not belonging to groups."

(17) Professor C. H. BECKETT: "A note on interest on reserve from items computed for the uniform report blank."

(18) Dr. A. E. YOUNG: "On asymptotic isothermic surfaces."

(19) Professor G. A. MILLER: "Some questionable terms and definitions used in elementary mathematics."

(20) Messrs. J. A. PARKHURST and F. C. JORDAN: "The photographic determination of star colors and their relation to spectral type."

(21) Mr. PHILIP FOX: "On the detection of the eruptive prominences on the solar disk."

(22) Mr. PHILIP FOX: "An investigation of the 40-inch objective at the Yerkes observatory."

(23) Mr. R. J. WALLACE: "The function of a color filter and of certain plates in astronomical photography."

(24) Professor G. E. HALE: "The vertical coelostat or 'tower' telescope of the Mt. Wilson solar observatory."

(25) Professor G. E. HALE and Mr. W. S. ADAMS: "Preliminary results of a comparative study of the spectra of the limb and the center of the sun."

(26) Professor FRANK SCHLESINGER: "A simple method for reducing spectrograms."

(27) Professor F. R. MOULTON: "On the probability of the near approach of two stars and on relative problems in the sidereal universe."

In the absence of their authors the papers by Professors Cairns, Halsted and Lovett and Dr. Ranum were read by title; that by Professor Schlesinger was presented by Mr. Fox. All the other papers in the above list were read by their authors. The following abstracts of those which were of mathematical interest bear numbers corresponding to those of the titles in this list. The abstracts of the others appeared in the report of this meeting published in *Science*, January 31, 1908.

5. Professor Lovett constructs a problem of three bodies possessing exact transcendental solutions defined by finite equations; these solutions reduce to the lagrangian solutions of the classic problem of three bodies when the ideal problem assumes the newtonian form. The paper is a part of an extended memoir which is not yet ready for publication.

12. By a generalization of the Hilbert theory of a quadratic form in an infinite number of variables, Professor Cairns gives a complete treatment for the integral equation

$$f(s) = \phi(s) - \lambda \int_a^b K(s, t)\phi(t)dt + x'g(s)$$

with the auxiliary equation

$$\int_a^b g(t)\phi(t)dt = c,$$

$f(s)$, $g(s)$, and c being given, $K(s, t)$ a generalized Green's function, and $\phi(s)$ the required function.

Application is made to the differential equation

$$p(x)\frac{d^2u}{dx^2} + \frac{dp}{dx}\frac{du}{dx} + q(x)u + \lambda'g(x) + \phi(x) = 0$$

with the auxiliary equation

$$\int_a^b u(x)g(x)dx = 0,$$

to a generalized Fourier development, and to the isoperimetric problem

$$\int_a^b F(y', y, x)dx = \text{minimum},$$

given

$$\int_a^b G_i(y', y, x)dx = l_i \qquad (i = 1, 2, \cdots, m).$$

13. The paper by Professor Halsted investigates the essence of inherent order, its foundation and genesis, and considers the uses of inherently ordered series in the attribution of factitious order to primarily unordered assemblages. It gives the solution of problems in arrangement and investigates the betweenness relations, linear and of more than one dimension. It gives applications to the theory of teaching, to arithmetic, and to geometry.

14. The method of approximation developed by Poincaré for obtaining the terms of higher order of the perturbative function depends uniquely upon certain singularities of this function. In the general case the singular points are given by algebraic equations of a high degree. The discussion relative to the actual admissibility of these points is often very delicate, but a criterion once established leads to the solution of the general case by extension of the results found by Poincaré, Hamy, Feraud, and Coculesco in certain restricted cases. With suitable numerical assumptions the case of small eccentricities and a small inclination of the orbits was carried out in detail by Dr. Reed, and the results to be found by varying the elements are indicated. The method is especially applicable to the problem of finding the values of the coefficients with small divisions in the perturbations of the small planets by Jupiter.

15. Professor Eiesland's paper contains a treatment of all the types of translation surfaces that are determined by a unicursal quartic in the plane at infinity. The most important result obtained is this: To a unicursal quartic with three real double points correspond ∞^3 types of translation surfaces of the form

$$(1)\quad A + Be^X + Ce^Y + De^Z + Ee^{X+Z} + Fe^{X+Y} + Ge^{Z+Y} + He^{X+Y+Z} = 0$$

where the coefficients satisfy the following identical relation:

$$EGAF = HDCB.$$

By a logarithmic transformation

$$x = \log X, \quad y = \log Y, \quad z = \log Z$$

this surface is converted into the cubic surface

$$(2) \quad A + Bx + Cy + Dz + Exz + Fxy + Gzy + Hxyz = 0$$

which is the exact analog of the surface

$$Ax + By + Cz + Dxy + Exz + Fzy = 0$$

obtained by Lie for the case of a degenerate quartic consisting of two conics.* It is the intention of the writer to study the surfaces (2) more in detail. The paper will be offered to the *American Journal of Mathematics* for publication.

16. All ordinary n-ary matrices (linear substitutions with non-zero determinants) form a group, and some singular matrices (with zero determinants) also belong to groups. If S is a singular matrix not belonging to any group, then if the number of zero roots of its characteristic equation is s, there always exists a positive integer $m \leqq s$ such that S^m belongs to some group. If m is the lowest such integer, then among the invariant factors of the characteristic determinant of S that correspond to zero roots there is at least one of order m and none of higher order. Dr. Ranum also showed that if S is of rank r, it is reducible to an ordinary $(n - s)$-ary partial matrix and $n - r$ singular, irreducible partial matrices, each of which is nilpotent, *i. e.*, has a power (exponent $\leqq m$) equal to zero.

17. By an agreement among the insurance commissioners and supervisory departments of a majority of the States of the Union, a detailed and comprehensive report of the year's business is required of life insurance companies on a form known as the union form report blank. In the "profit or loss" account of this sheet there is an item "interest required to maintain reserve." The net income from investments is exhibited and from these items the "profit or loss" from interest earnings is shown. There is a discrepancy between the mathematical theory involved in the fundamental calculations, and the policy contract itself which leaves some latitude and consequent confusion. The purpose of Professor Beckett's paper is to set forth this problem and find a solution that can be checked from the other

*See Lie-Scheffers, Berührungstransformationen, vol. 1, pp. 367 and 410.

items required to be computed and exhibited in the report, in the general interest of a fair understanding and uniformity. All life insurance calculations are based upon the theory that premiums are paid at the beginning of the insurance year, and death claims paid at the end of the year. All modern life insurance contracts provide that payment shall be made upon satisfactory proofs of death, or in a few cases within 60 days thereafter.

The death strain or "mortality cost" in any case is the amount at risk multiplied by the probability of dying, *i. e.*, $C_{x+m}(S - V_{x+m})q_{x+m}$. The mean reserve which appears in the report blank is $\frac{1}{2}(P_x + V_{x+n} V_{x+n+1}) = m_{x+n+\frac{1}{2}}$. The net cash value of the policy at the end of any year, or terminal reserve, is the fund which together with the net premiums to be received in the future is the exact mathematical equivalent of the obligation incurred by the company to pay the contract.

In any individual case, interest $3\frac{1}{2}\,\%$,

$$1.035\,V + P_x(1.035) = V_{x+n+1} + C_{x+n},$$

$$M_{x+n+\frac{1}{2}} = \frac{V_{x+n} + P_x + V_{x+n+1}}{2}, \quad V_{x+n+1} = 2M_{x+n+\frac{1}{2}} - V_{x+n} - P_x,$$

$$V_{x+n} = \frac{2M_{x+n+\frac{1}{2}} - V_{x+n} - P_x + C_{x+n+1} - P_x(1.035)}{1.035},$$

$$V_{x+n} = \frac{M_{x+n+\frac{1}{2}} + \frac{1}{2}C_{x+n+1} - P_x(1.0175)}{1.0175}.$$

Let

$$V_{x+n} + P_x = R, \quad R = \frac{M_{x+n+\frac{1}{2}} + \frac{1}{2}C_{x+n+1}}{1.0175}.$$

Interest for the first half year

$$R(.0175) = \frac{.0175}{1.0175}\left[M_{x+n+\frac{1}{2}} + \tfrac{1}{2}C_{x+n+1}\right].$$

Let $V_{x+n-1} + P_x = K$.

Interest for the second half year

$$K(.0175) = \frac{.0175}{1.0175}\left[M_{x+n+\frac{3}{2}} + \tfrac{1}{2}C_{x+n+2}\right].$$

If from these sums there be deducted $3\frac{1}{2}$ per cent. of the mean net deferred premiums, the result gives the interest required to maintain reserve in terms of other items which are reported in the blank. This is a close check and true in theory, but not quite true in actual practice and contract. In case a death claim falls in the early part of the insurance year on a contract that is young, the loss is conspicuous. The only way to make a perfect agreement between theory and contract would be to recompute all net premiums and all other tables involved in the calculations. Upon the new assumption that claims are paid immediately, this is possible but not practical, as it would cause great confusion, and since it could not be retroactive, it would take a long period of years to have all forms in conformity to this practice. In the meantime great confusion would prevail. Should the company earn only this guaranteed rate of interest upon net premiums and reserves, there would be a deficiency of approximately .0035 per cent. on a mixed business of paid-for insurance, endowment, and limited payment forms.

Claims cannot be paid actually at the moment of death. Statistics show that the actual necessary lapse of time averages one month and that on an average the loss of interest upon experienced mortality funds paid out is five months. The best practical adjustment would then be to discount this fund upon this five-month basis and require the company to make good the deficiency out of excess earnings over the rate assumed in the contract.

18. Surfaces characterized by having isothermal asymptotic lines and isothermal lines of curvature Dr. Young has called "asymptotic-isothermic," and in a paper read before the Chicago section of the AMERICAN MATHEMATICAL SOCIETY, March 30, 1907, he outlined a general method for the determination of these surfaces and discussed a particular solution of the problem which led to certain surfaces whose first fundamental forms, when referred to lines of curvature, come under the more general form

$$\overline{ds}^2 = \frac{(u+v)^k}{(u-v)^l}\left(\frac{du^2}{U} + \frac{dv^2}{V}\right),$$

where k and l are constants, and U and V are functions of u and v respectively. He continues the discussion in this paper and determines all surfaces which have the above expression for their first fundamental form when referred to lines of curvature, and also all isothermic surfaces which have the same spherical representation of their lines of curvature as these. All of the new surfaces are asymptotic-isothermic, with the exception of one class composed of an infinity of surfaces.

19. The term division has two distinct meanings in elementary mathematics.* According to one of these, it implies the operation of finding an integral quotient and an integral remainder, while according to the other it implies the finding of a number which multiplied into the divisor produces the dividend. While only the latter is the inverse of multiplication, yet it is customary to speak of division as an inverse operation without specifying which of the two commonly accepted definitions of the term is meant. A very common definition of multiplication is the performing upon the multiplicand the same operation as that which is performed upon unity to get the multiplier. The vagueness of this definition follows directly from the fact that 4 may be obtained by doubling unity and squaring the result, yet multiplying by 4 does not generally mean doubling the multiplicand and squaring the result. Such vague definitions are contrary to the very essence of mathematics and hence should be avoided.

Especial stress was laid by Professor Miller upon the fact that dividing by 0 should be banished from elementary mathematics. The so-called indeterminate forms are really meaningless forms and it is questionable whether one should speak of evaluating such a form. As $uv = 0$ has not always for its locus the combined loci of $u = 0$ and $v = 0$, the rule relating to this case should be stated with the necessary restrictions. The fact that the last letters of the alphabet are used both for variables and for unknowns in elementary algebra has led some authors to speak of these two generally distinct concepts as if they were identical. This is the more unfortunate since the concept of a variable is continually playing a more fundamental rôle in elementary algebra. The paper has been offered to *School Science and Mathematics* for publication.

27. The problem of Professor Moulton's paper is to find the possible rôle that the near approach of the stars to one another

* Cf. Encyclopédie des Sciences mathématiques, vol. 1 (1904), p. 43.

may have played in sidereal evolution. The answer to this problem depends upon the extent of the sidereal universe, the number of stars in it, and the character of their motion. The discussion leads to the conclusion that the relatively near approaches of the stars have probably been an important factor in stellar evolution. This paper will be published by the Carnegie Institution of Washington.

The next regular meeting of the association will be held at Johns Hopkins University under the presidency of Professor T. C. Chamberlin, of Chicago University. A summer meeting will be held at Dartmouth College, beginning the last Monday in June. Professor C. J. Keyser, Columbia University, was elected vice-president and chairman of Section A, and Professor G. A. Miller, University of Illinois, was reelected secretary. The former election is for one year and the latter for a period of five years in accord with the general rules of the association. A resolution of the council which is of especial interest to the various scientific societies is that sectional committees may dispense with a sectional program whenever an affiliated society covering the same scope meets with the section. This resolution was reaffirmed to avoid duplications of programs and to secure more hearty cooperation between the scientific societies and the association.

G. A. MILLER,
Secretary.

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SHORTER NOTICES.

Memoir and Scientific Correspondence of the late Sir George Gabriel Stokes, Bart., Sc.D., etc. Selected and arranged by JOSEPH LARMOR, D.Sc., LL.D., Sec. R.S. Two vols. 8vo. Cambridge, at the University Press.

THE history of mathematics and physics during the latter half of the nineteenth century is inseparably bound up with the career of Sir George Gabriel Stokes. As an investigator of the highest rank himself and as one who left a permanent mark on his own science, the story of his life and career is a necessary part of the development of the subject. But it is much more than this. His position as secretary of the Royal society