THE THEORY OF ELECTRICITY.

Theorie der Elektrizität. Von M. ABRAHAM. Zweiter Band: *Elektromagnetische Theorie der Strahlung.* Leipzig, B. G. Teubner, 1905. x + 404 pp.

THE first volume of Abraham's treatise on electricity, which was written as a revision of Föppl's earlier book on the same subject, has already been reviewed in the BULLETIN.[*] At the close of that notice the statement was made that "The second volume will be awaited with impatience." The volume appeared within a few months, and by this time the author would probably feel justified in concluding that however much the reviewer may have been impatient for the appearance of the volume, he was not particularly impatient to review it. This very tardiness will, however, be of no inconsiderable aid in writing a comment on the book.

Abraham's second volume deals with the theory of electrons. It is divided into two parts of which the first treats the field and motion of individual electrons and the second the electromagnetic phenomena in ponderable bodies. Although physical science has taken long strides in the past forty months along the path of electron theory, so that now the electron and its major properties must be considered by all as firmly intrenched facts of physics instead of grudgingly acknowledged theories, and although J. J. Thomson in a recent number of the *Philosophical Magazine* has contributed much in the way of enlightenment to our hitherto very vague notion of the nature of positive electricity, nevertheless comparatively little of that which has been accomplished in addition to what was known at the time of the publication of this volume can as yet be construed as offering very material aid in constructing or revising the mathematical theory of electricity from the electronic point of view. What the electron does in the large — a large which may be measured in small fractions of a $\mu\mu$ — is tolerably well known; but what its characteristics and behavior may be within a distance of one or two 10^{13}ths of a centimeter from its center is still much of an eleusinian mystery.

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[*] Volume 11, pp. 383–387 (April, 1905). The date at the end of the review should be February, 1905, not February, 1904. —We may note that a new edition of this first volume has just appeared.

The uninitiated may not be aware that there are at least four distinct electrons which have reached a considerable mathematical development. The first of these may be called the Larmor electron, inasmuch as the best exposition of its properties is found in Larmor's Aether and Matter. It is a mathematical point endowed with a finite charge of electricity which creates a certain strain or "beknottedness" in the surrounding ether. Of course nobody, Larmor least of all, would maintain that the real physical electron was thus devoid of extension in space. The conception is merely preliminary and is adopted for the purpose of obtaining mathematical simplicity. The second electron may be called the Abraham electron. It is discussed in detail in the book under review and previously to the publication of this book it had already received great development at the hands of the author. Its dimensions are finite and its configuration is spherical with a radius r which probably lies within the limits $10^{-13} < r < 2 \cdot 10^{-13}$. It is rigid. As Lorentz is so nearly the father of all mathematical electron theory, it is perhaps unfair to designate any particular electron by his name. The particular electron, however, which is called the Lorentz electron is also of finite dimensions. When at rest it is spherical, but when animated with a motion of translation it becomes an ellipsoid of revolution with the axis of revolution shortest and directed along the line of motion. If r be the radius of the sphere at rest and if β be the ratio of the velocity of the electron to the velocity of light, the semiaxes of the ellipsoid perpendicular to the line of motion remain equal to r, whereas the axis along the path becomes $r(1 - \beta^2)^{\frac{1}{2}}$. This introduces a shrinking along the line of motion and was adopted for the purpose of explaining the troublesome Michelson-Morley experiment. Finally there is the nearly simultaneous creation of Bucherer and Langevin — an electron which shortens in the direction of motion but expands in the perpendicular direction by an amount sufficient to make the volume of the ellipsoid equal to the volume of the original sphere; the new semiaxes are $r(1 - \beta^2)^{-\frac{1}{6}}$, $r(1 - \beta^2)^{-\frac{1}{6}}$, and $r(1 - \beta^2)^{\frac{1}{3}}$.

Fortunately the greater part of the mathematical theory of each of these four electrons is common to the other three. It is only in effects which may be called of the second order that radical differences occur, and even then the differences are not always great. Two very delicate experiments, one the Michelson-Morley experiment already referred to and the other the

Kaufmann experiment, have perhaps been the chief tests of the applicability of the different theories. On the hypothesis that an electron is a charged particle of finite size, the particle in motion will possess an inertia of electric origin which will increase with the velocity and become infinite when the velocity approaches that of light. The increase is very slow at first and scarcely becomes appreciable until the fraction β has passed the the value 0.5. If it be assumed that the entire inertia of the electron is of electric origin — a fact which seems to be clearly indicated by Kaufmann's earlier experiment — it follows that the mass of the electron, whether the transversal or the longitudinal mass, is a subject for purely electromagnetic experiment and theory.

If m denote the transversal mass and m_0 be the value of this mass when $\beta = 0$, the mass for any velocity may be written as $m_0\Phi(\beta)$, where $\Phi(\beta)$ is a function of β which has different expressions according to the different theories, namely,

$$\Phi(\beta) = \frac{3}{4}\frac{1}{\beta^2}\left(\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right) \text{ for Abraham's electron,}$$

$$\Phi(\beta) = (1-\beta^2)^{-\frac{1}{2}} \text{ for Lorentz's electron,}$$

$$\Phi(\beta) = (1-\beta^2)^{-\frac{1}{3}} \text{ for Bucherer's electron.}$$

Since the appearance of Abraham's book, Kaufmann has carried out an extended series of experiments * from which he infers that either the Abraham or the Bucherer electron represents the experimental facts within an error which is less than the experimental error, whereas Lorentz's electron gives calculated values which depart from the observed values by an amount considerably in excess of the experimental errors. Hence, so far as these particular experiments are concerned, it appears that there are still two types of electron to dispute the field.

Another lengthy investigation † which has appeared since Abraham's book is due to Poincaré. Instead of being experimental it is purely mathematical and based on the Michelson-Morley experiment and the principle of relativity. Here the

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* Printed in the *Annalen der Physik*, volume 19 (1906). The range of values of β is about $0.55 < \beta < 0.97$. It is really remarkable that two theories should agree within the small error of two or three per cent. over such a wide range.

† Printed in the *Rendiconti di Palermo*, volume 21 (1906).

theory of groups of transformations does valiant service. The principle of relativity is the postulate that there never can be obtained a method for distinguishing absolute motion; and this certainly accords with past experiments on the subject, which have constantly afforded merely negative results. Poincaré examines the consequences of the assumptions of Abraham, Lorentz, and Bucherer. He finds, among other things, that if an electron is constrained by a relation between its three axes and if no other forces not of electromagnetic origin act upon the electron, then the Bucherer-Langevin hypothesis is the only one admissible. On the other hand this hypothesis does not admit the principle of relativity. Lorentz had previously established this result in a different manner. Poincaré finds also that the only hypothesis which is in accord with the principle of relativity is that of Lorentz; but here it is necessary to introduce forces of order other than electromagnetic, and these forces are derivable from a potential which is proportional to the volume of the electron.

It is needless to remark that in a subject so replete with hypotheses and difficulties as the electron theory there is always a possibility that something has been overlooked and that some day another interpretation may be available which will vitiate present conclusions: but Kaufmann is a particularly careful and habile experimenter and Poincaré is no less an astute mathematician, and it is interesting to combine their conclusions even if to-morrow may call for a revision of the inference. From Kaufmann's experiment let us conclude to throw out the Lorentz electron. Poincaré's work then shows that the principle of relativity cannot subsist. It is certainly more satisfactory philosophically and scientifically to be left with the hope that some time we may be able to distinguish absolute motion than to feel that we shall in nowise be able to do so. Again making an appeal to Poincaré's investigation, we may infer that the Bucherer-Langevin electron is the best to adopt inasmuch as it does not necessitate the introduction of other forces than those of electromagnetic origin and those involved in the relation between the axes of the ellipsoid of revolution. Surely students of electricity will prefer the simple expression of $\Phi(\beta)$ in this system to that given by Abraham, and to any follower of Maxwell the supposition that the volume of the electron is constant must be a source of considerable consolation.

The first chapter of Abraham's book is on the physical and mathematical foundation of the electron theory. The treatment is excellent. The historical notes, the collection of evidence culminating in the irresistible inference of the existence of electrons and their simple relations to problems of electromagnetism, the insistence on the numerical values of the fundamental magnitudes, and the systematic classification of radiation make greatly for the ease and contentment of the reader. No sooner are the fundamental equations set up than the author proceeds to develop the ideas of electromagnetic momentum and moment of momentum. This parallel to ordinary mechanics aids in the comprehension of the text and allows the author to emphasize the differences between the laws of electricity and those of matter. The question of the inequality of action and reaction, which at first caused considerable difficulty to some investigators, is treated with a detail and clearness which leave nothing to be desired. The important results are stated here and throughout the book in spaced type so as to catch the eye. The integration of the equations to obtain the formulas for the potentials is accomplished by a method due to Abraham and based on the familiar method given for a special case in Weber's Partielle Differentialgleichungen. This is the only complicated piece of analysis in the chapter.

The second chapter treats the radiation of waves from a point charge in motion and is consequently of importance in the theory of optics. It will not be amiss to call especial attention to the words point charge. The author takes pains to indicate what portions of his theory are independent of his particular assumption of rigid spherical electrons. This is of high value to the reader, for whom it is a matter of importance to have those portions of the work which would be true on any of the current hypotheses concerning electrons separated from the consequences due to Abraham's special electron. This chapter, then, is concerned with what happens at a considerable distance from the electron. The model for a radiating source of light is the electric dipole, consisting of a fixed positive charge and a moving negative charge. The loss of energy through radiation is computed, and not merely computed as a formula but actually turned into numbers from available experimental data. The emphasis laid upon the order of numerical magnitude of the quantities involved in a calculation will almost inevitably serve to differentiate a true mathematical physicist from a mathematician who merely looks to physics for problems to which to apply his analysis. In this chapter the Zeeman effect naturally comes in for a detailed discussion. The field due to an electron moving uniformly or with an acceleration is taken up and applied to the theory of a moving source of light. The questions of longitudinal and transverse vibrations are handled separately, and the matter of the reaction of the radiation on the source is not overlooked. It is a happy idea of the author's not to be in any haste in passing over these fundamental points.

The mechanics of electrons is the subject of the third chapter. There is an introduction in which the reasons for assuming that the mass of the electron is wholly of electromagnetic origin are outlined and in which the relation of the electromagnetic cosmos to the mechanical cosmos discussed by Hertz in his Mechanics is set forth. The author then gives a detailed argument in favor of his rigid spherical electron. His aim is to construct the world on a purely electromagnetic basis, and the assumption of any other hypothesis concerning the electron appears to him to necessitate the introduction of enormous elastic forces of non-electromagnetic origin. This seems scarcely conclusive, especially in view of Poincaré's recent memoir. It is well known that when a charged sphere is in motion the lines of force, alias the Faraday tubes, are drawn toward a plane through the center of the sphere and perpendicular to the path. Why this might not quite naturally result in the Bucherer-Langevin deformation is hard to see. In fact it would seem that when an electron is in motion it would require enormous forces of rigidity which were not of electromagnetic origin to preserve the spherical shape of the electron. If the author were to reprint his book now, he would doubtless give this question a more thorough treatment.

In this chapter on the mechanics of electrons Abraham again keeps close enough to ordinary mechanics to speak of momentum, moment of momentum, moment of inertia, and the lagrangian function. This part of the work is as interesting as any and is highly to be recommended for its pedagogic excellence, especially in view of the fact that when the book was written it had many persons to convince as well as to instruct. The matter of electromagnetic mass, whether transversal or longitudinal, is expounded in all detail and is set into relation with the lagrangian function. The Lorentz electron is treated and its relation to the author's is indicated. The discussion ends with the statement that Lorentz had shown that his electron satisfied the (earlier) Kaufmann experiments about as well as Abraham's and with the expression of the hope that further refinements of the experiment would serve to differentiate between the two. That hope was soon to be realized in favor of the author. Unfortunately no word is mentioned concerning Bucherer's electron. This is a serious omission, although perhaps unavoidable, inasmuch as Bucherer's book * had appeared only some six or eight months before Abraham's. The chapter concludes with some interesting though partly speculative investigations such as the discontinous motion of electrons (useful' in the explanation of Röntgen rays), the force in the interior of an electron, and the uniform motion of an electron with a velocity greater than that of light.

The author then comes to the second part of his work, that on electromagnetic phenomena in ponderable bodies. This he divides into two chapters, the first on stationary bodies, the second on moving bodies. After a preliminary discussion of what is to be understood by the term physically small, he introduces the method of averages to derive the ordinary equations of the electromagnetic field in bodies at rest. The dispersion of electromagnetic waves and the connection with the index of refraction, the magnetic rotation of the plane of polarization, the question of magnetization, and electric conduction in metals are treated from the point of view of electron theory. The name of Drude occurs frequently in these pages, but we do not find that of J. J. Thomson. To be sure, a great deal of Thomson's work † in this particular direction had not appeared at the time of writing and considerable more of it might have been thought to be still in too speculative a state and insufficiently capable of presentation in a form assimilable with the rest of the book; this, however, was hardly true of all of it. Other omissions, such as the theory of optical rotation in crystals and solutions and the explanation of the Peltier effect, may be explained by the simple statement that even now electrons have not enabled us to account for all electrical phenomena. As a general criticism, though not a severe one, we may say

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* Mathematische Einführung in die Elektronentheorie. B. G. Teubner, 1904, 148 pp. This is perhaps the best short account of an electron theory. On account of its brevity it is more exclusively mathematical and consequently of greater average difficulty than the work under review.

† We may mention his book Electricity and Matter, Charles Scribner's Sons, 1904 ; 162 pp. Also the extremely recent Corpuscular Theory of Matter, Constable, 1907 ; 180 pp. The latter we have not yet seen.

that it would have added to the interest of the book if the author had gone more into detail from time to time as regards the things which were yet to be accounted for by the theory instead of confining himself so exclusively to matters which he actually was in a position to analyze. In conclusion to the chapter there is a lengthy discussion of the mathematics of wireless telegraphy.

The concluding chapter on moving bodies commences with the careful and critical derivation of the equations of the field followed by a discussion of Fizeau's experiment. The crucial nature of this experiment for deciding between Lorentz's optics and Hertz's is brought out. There follow sections on the pressure of radiation on surfaces in motion, whether they be black or reflecting. It would have been possible to add somewhat to the interest of this question by entering upon Poynting's applications of the results to cosmical speculations. In the section on the temperature of radiation we find the laws of Kirchhoff and Wien. Here the author, as in so many other places, goes into the matter numerically. Next follows the treatment of the Michelson-Morley experiment and its crucial evidence against a stagnant ether or in favor of a contraction along the path. This leads to a presentation of the optics of Lorentz and Cohn; and with that the book comes to a close except for the extensive index.

From what has been said it cannot fail to appear that we have in this treatise a work which deals with the most fundamental questions of physics and sets them into relation with the latest developments of theory and experiment. In only a few places is the analysis complicated, and everywhere there is an abundance of physical data which are frequently worked out to their numerical consequences. To an unusually large extent the book represents the work of individual investigation on the part of the author. It could not have been written as a compilation from the accomplishments of others. Whether the Abraham electron shall persist or be cast aside, the greater portion of the present volume will remain, and most of the rest will have to be changed but little. Clearly those who impatiently awaited the appearance of this second volume have not been disappointed in their expectations of it.

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