THE FIRST REGULAR MEETING OF THE SOUTHWESTERN SECTION.

THE first regular meeting of the Southwestern Section of the AMERICAN MATHEMATICAL SOCIETY took place in St. Louis, Mo., on the 30th of November, 1907. About fifty persons, including the following members of the Society, were present:

Professor L. D. Ames, Mr. Charles Ammerman, Professor Florian Cajori, Professor A. S. Chessin, Professor C. E. Comstock, Brother Constantius, Professor S. C. Davisson, Professor G. R. Dean, Professor G. W. Droke, Dr. Otto Dunkel, Professor B. F. Finkel; Professor G. B. Halsted, Mr. W. W. Hart, Professor H. C. Harvey, Professor E. R. Hedrick, Dr. Louis Ingold, Dr. G. O. James, Professor O. D. Kellogg, Professor G. A. Miller, Professor H. B. Newson, Professor J. H. Scarborough, Professor J. B. Shaw, Mr. R. L. Short, Professor H. E. Slaught, Dr. Clara E. Smith, Mr. H. P. Stellwagen, Mr. E. H. Taylor, Professor C. A. Waldo, Professor B. M. Walker, Dr. Paul Wernicke.

The meeting was called to order by Professor Hedrick at 10:30 A. M., at the McKinley High School. The morning session adjourned at 12:30 P. M. The afternoon session was called to order at 3 P. M., at Washington University. The reading of papers was preceded by a short business meeting. Professor Chessin (chairman), Professor Kellogg (secretary) and Professor Newson were elected to serve on the program committee for the ensuing year. The University of Kansas was unanimously selected as the next meeting place of the Southwestern Section.

The following papers were read:

(1) Dr. P. WERNICKE: "On Euler's tactical '36 officers' problem."

(2) Dr. P. WERNICKE: "Extension of the map-color theorem to 3 dimensional space."

(3) Professor W. F. OSGOOD: "On the differentiation of definite integrals."

(4) Professor F. CAJORI: "Notes on the history of the slide rule."

(5) Dr. LOUIS INGOLD: "Note on areal cross ratios."

(6) Dr. G. O. JAMES: "A relation connecting aberration and parallax" (preliminary report).

(7) Professor J. B. SHAW: "A new graphical method for quaternions."

(8) Professor E. R. HEDRICK: "On a definition of the jacobian" (preliminary report).

(9) Professor A. S. CHESSIN: "On an integral appearing in photometry."

(10) Professor O. D. KELLOGG: "Real roots of an algebraic equation."

(11) Dr. LOUIS INGOLD: "Note on a connection between algebraic invariants and the invariants of a differential form."

(12) Professor H. B. NEWSON: "On the resultant of two collineations."

(13) Professor G. A. MILLER: "On the holomorph of the cyclic group of order p^m."

(14) Professor E. W. DAVIS: "Colored imaginaries on a cubic."

In the absence of the authors, Professor Osgood's paper was read by Professor Hedrick and Professor Davis's paper was read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Euler and Cayley have given some attention to the problem of arranging in a square of n rows and columns n^2 officers of n different regiments, the n of each regiment being of n different ranks. Every rank and regiment is to be represented in each row or column. Such arrays are possible unless $n \equiv 2 \pmod 4$. No proof is extant of their impossibility in this particular case. Dr. Wernicke attempted such a proof on the principle that the four classifications involved (viz. into rows, columns, officers' rank and regiments) are tactically equivalent. Hence applications become possible to more complex problems involving an interference of classifications.

2. Dr. Wernicke defines a "three-dimensional map" to be a division of three-dimensional space by any number of surfaces meeting in lines only. The latter meet in points only, which we call the vertices of the map. The portions of the lines extending from any vertex to the consecutive one are the edges; the portions of the surfaces bounded by the edges, the partitions; the portions of space partitioned off, the cells or districts of the map. If a map contains neither multiply-connected cells nor partitions, its cells may be filled with nine different gases in such a way

that no partition separates cells holding the same gas. The method of proof is similar to that of the corresponding four-color theorem of plane maps.

3. In this paper Professor Osgood gives an account of the existing proofs of the formulas for differentiating a definite integral with respect to a parameter, followed by a critique of these proofs. He then gives a new proof based on Green's theorem, which is free from some of the objections which may be brought against the older proofs. The paper will be offered to the *Annals of Mathematics*.

4. Charles Hutton attributed the invention of the straight-edge slide rule to Edmund Wingate, but gave no references to Wingate's publications. Augustus De Morgan attributed the invention to William Oughtred and denied that Wingate ever wrote on the slide rule. Professor Cajori was able to quote from one of Wingate's works of 1628 and to show that Wingate was the first inventor. The "runner," often attributed to Mannheim (1850), was shown to be the invention of I. Newton and E. Stone. Probably the first inversion of a logarithmic line is found in Everard's slide rule, used in England in gauging during the latter part of the seventeenth and in the eighteenth century. Newton's and Stone's adaption of the slide rule to the solution of numerical equations was explained. Finally, the time of the introduction of the slide rule into the United States was considered.

5. Dr. Ingold considers the ratio of areas of triangles formed from six points, A, B, C, P, Q, R, in a plane. The double ratio $[PA, BC, QR]$ is defined to be

$$\frac{PBC}{ABC} \div \frac{PQR}{AQR},$$

which can be proved to be invariant under projection. The possible double ratios are seen to be equal in sets of eight; there remain therefore ninety which are distinct. From identities such as $ABC.PQR = PBC.AQR + QBC.PAR + RBC.PQA$ a number of relations are easily established among the different double ratios, so that not more than four of the ninety are independent, the others being expressible in terms of these.

6. In any system of spherical coordinates the aberrational corrections may be computed from the corresponding formulas for the parallaxes by differentiating these formulas with respect to the time, treating the angular coordinates as constant. Dr. James obtains the following results:

$$\Delta A_a = \frac{d}{dt}\left(\frac{rP}{\cos B - Q \sin B + R \cos B}\right)$$

$$V\Delta B_a = \frac{drQ}{dt} - \frac{drP}{dt} \sin B \tan \frac{\Delta A_a}{2}.$$

7. Any quaternion $q = w + xi + yj + zk$ may be written in the form $q = w + xi + (y + zi)j$. We may represent a quaternion graphically therefore by representing $w + xi$ and $y + zi$ by the ordinary method of representing complex numbers, $w + xi$ being drawn on one plane and $y + zi$ on a second plane, called the j-plane. For convenience we may divide an ordinary plane into four quadrants, the first quadrant being the plane of $w + xi$, the third the quadrant of $y + zi$. The second and fourth quadrants, by orthogonal projections, furnish also representations for $x + yk$, and $w + zk$. The product and the sum of two quaternions may be found graphically by methods similar to those in use in the theory of complex numbers. The theory of two independent complex variables finds a graphical representation here also. Finally the descriptive geometry of four dimensional space and the quaternion algebra as applied to four-dimensional space come into intimate contact, and analytic processes may be easily interpreted in geometric procedures, or conversely. The paper will be offered to the *Transactions*.

8. In this paper, Professor Hedrick offers a modification of the definition of the jacobian suggested by Jordan (Cours d'analyse volume 1, § 148) and by Porter (*L'Enseignement mathématique*, volume 9, part 4, page 272). The jacobian is defined as the limit of the quotient of two areas, one of which is a right triangle in the original plane. This limit exists whenever the ordinary jacobian exists, and á fortiori whenever the limit used by Jordan and Porter exists. Certain advantages are pointed out.

9. Professor Chessin's paper appears in full elsewhere in the present number of the BULLETIN.

10. By a theorem of Professor Van Vleck (*Annals of Mathematics*, July, 1903), an algebraic equation with real coefficients $c_0 + c_1x + c_2x^2 + \cdots + c_nx^n = 0$ will have all or all but one of its roots (according as n is even or odd) imaginary if all the members of a certain set of determinants are positive. Professor Kellogg deduces from Descartes's rule of signs a necessary condition that all the roots of such an equation be real; namely that all the determinants $c_{j-1}c_{j+1} - c_j^2 < 0$ ($k < j < n$, where c_k is the first coefficient different from 0). A second, sharper condition, of which the first is a consequence, is that none of the quadratics

$$\begin{vmatrix} x^2 & x & 1 \\ c_{j-1} & c_j & c_{j+1} \\ c_{j+1} & c_{j+1} & c_{j+2} \end{vmatrix} = 0 \qquad (k \leqq j < n, \;\; c_{-1} = c_{n+1} = 0)$$

has real roots. The paper will appear in the January number of the *Annals of Mathematics.*

11. In Dr. Ingold's second paper a quadratic differential form D with constant coefficients is considered. The differential parameters of such a form which involve no functons except algebraic forms $b, c, \cdots$ are concomitants of these forms and of the quadratic algebraic form a having the same coefficients as D. Conversely, any concomitant of the system $a, b, c, \cdots$, when multiplied by a power of the discriminant of D, becomes a differential parameter of D. Relations among the differential parameters of D, when thus specialized, yield relations among the concomitants of the system $a, b, c, \cdots$.

12. Professor Newson's paper seeks to determine invariants of collineations. As one such is found a certain cross ratio, and it is shown that this cross ratio for the product of two collineations is a simple function of the two corresponding cross ratios for the two collineations determining the product.

13. The paper by Professor Miller is a continuation of his article entitled "On the holomorph of a cyclic group" which was published in the *Transactions*, volume 4 (1903), page 151. While the holomorph of a cyclic group plays a fundamental role in many group theory considerations, it also involves all questions in regard to the exponent to which numbers belong with respect to a given arbitrary modulus m. For instance,

the numbers which Epstein, in his recent article published in the *Archiv der Mathematik und Physik,* calls primitive roots of m are those which correspond to the operators of highest order in the group of isomorphisms of the cyclic group of order m, and hence the determination of the number of such primitive roots is a very special case of the determination of the number of operators of a given order in an abelian group. In the present paper special attention is paid to the group of isomorphisms of the holomorph of the cyclic group of order 2^m, and one of the most important results is stated as follows: The group of isomorphisms of the holomorph of the cyclic group of order 2^m is the direct product of the group of order 2 and the group of cogredient isomorphisms of the double holomorph of the cyclic group of order 2^m. This paper will appear in the *Transactions.*

14. In this paper Professor Davis studies the connection between his theory of colored imaginaries and the pole and polar theory of the cubic curve. If $f(x, y, z) = 0$ is the equation of the cubic, $\Delta'(x, y, z)$ the first polar of x', y', z' with regard to $f(x, y, z)$, while Δ'' is the polar of x'', y'', z'', then when we write $x = x' + ix''$, etc., we get $f(x', y', z') = \Delta'(x'', y'', z'')$ and $f(x'', y'', z'') = \Delta''(x', y', z')$. It is upon the basis of these two equations that the paper is built.

O. D. KELLOGG,
Secretary of the Section.

NOTE ON THE COMPOSITION OF FINITE ROTATIONS ABOUT PARALLEL AXES.

BY PROFESSOR ALEXANDER ZIWET.

1. IT is well known that the succession of two finite rotations of a rigid plane figure in its plane (or, what amounts to the same, of a rigid body about parallel axes), say a rotation of angle θ' about a point O' followed by a rotation θ'' about O'', is equivalent to a single rotation of angle $\theta = \theta' + \theta''$ about a point O. The center O is found as the intersection of the lines obtained by turning $O'O''$ about O' through an angle $-\frac{1}{2}\theta'$ and $O''O'$ about O'' through $+\frac{1}{2}\theta''$.

As a clockwise rotation of angle ϕ is equivalent to a counter-clockwise rotation of angle $2\pi - \phi$, the angles of rotation can