

the hypergeometric equation. We are thus led directly to the theory of automorphic functions. Space is found for a short discussion of the dreieck functions, *i. e.*, functions which map the half plane on a triangle whose sides are circles.

We hardly need to add in closing that we recommend Herr Maurer's book most heartily to the student. Even the instructor will, we doubt not, find fresh inspiration in perusing its pages, and find here and there a mode of treatment which he will be tempted to incorporate into his own lectures.

JAMES PIERPONT.

*Elliptische Funktionen.* Von HEINRICH BURKHARDT. Zweite, durchgesehene und verbesserte Auflage mit zahlreichen Figuren im Text. Leipzig, Veit and Company, 1906. 373 + xvi pp.

THE present edition is essentially a reproduction of the first, except that here and there a proof has been improved or a typographical error has been corrected. Numerous friends of the book have sent the author lists of errata. Thanks to their cooperation, the author hopes the formulas are now entirely reliable. This is certainly a most important feature in a subject which almost suffers from its inexhaustible wealth of formulas.

For a detailed account of the contents and tendencies of this superior work, the reader may consult an extended review in this BULLETIN for July, 1900, pages 452-463.

JAMES PIERPONT.

*Quadratic Forms and their Classification by Means of Invariant Factors.* By T. J. P. A. BROMWICH. Cambridge University Press, 1906. viii + 100 pp.

THE theory of elementary divisors, with which this book deals, is one of the most useful and perfect of algebraic theories. Although it is now nearly forty years since Weierstrass's fundamental paper was published, no treatment of the subject appeared in English until the year 1904, when a brief discussion was included in Mathews's revision of Scott's *Determinants*. This treatment is far from being suited to the needs of one wishing to penetrate for the first time into the theory, and the same is true of the only treatise on the subject which exists in any language, that of Muth. The appearance of Mr. Bromwich's book is therefore to be hailed with satisfaction as affording the

first opportunity for those who prefer to use the English language to become familiar with this subject without an undue expenditure of labor.

The author replaces Weierstrass's term *elementary divisor*, which is now in universal use, by the term *invariant factor*. This is rather a high-handed proceeding and would seem to require justification. Weierstrass's term is a suitable one and admits of satisfactory translation into English; so that it is hard to see what the author's purpose can have been in making the change.

The author does not develop the theory except so far as it relates to the problem of the classification of quadratic forms; and, in fact, the whole book is directly connected with this problem, and with its geometrical interpretations in the cases of two, three, four, and to a less extent, five or six variables. The constant reference to geometrical illustrations, and the introduction of simple geometrical cases at an early stage form an admirable feature of the book. The author has elaborated a method of treatment, due in its essential features to Kronecker, which has the advantage over Weierstrass's original method of avoiding the necessity for the proof of certain difficult theorems concerning determinants.

In one respect the treatment is very complete since it covers not merely the cases originally considered by Weierstrass, but also the *singular case* first treated by Kronecker.

The method of presentation is thoroughly elementary, and a reader with some slight familiarity with the use of trilinear coördinates should have no difficulty in working through the little volume from cover to cover, and, in the end, really knowing what elementary divisors are and how they are to be used. After having said this, it may seem strange to add that the main results of the theory are not sufficiently emphasized, sometimes hardly even explicitly stated. This, however, is the case, and it makes the book of very much less value than it might otherwise have been to persons who wish to get at the main lines of the theory without going through all the details of the proofs and of the numerical examples which are more or less mixed in with them. The volume has no index.

MAXIME BÔCHER.