proves this theorem * without giving reference to the less general theorem proved by Loewy which is mentioned at the end of my report on the groups of an infinite order.† Another test given in this paper for the finiteness of a linear substitution group on a finite number of symbols is that it contains a finite number of distinct sets of conjugate substitutions.

Quite recently Loewy has investigated the groups of linear homogeneous substitutions which are of the type of a finite group and gave a complete development of their theory.‡ Dickson has recently published two papers in which he considers for the first time the problem of representing a given finite group as a linear congruence group.§ He points out that the only one of the different expositions of Frobenius's theory of group characters mentioned above which may be utilized in the construction of a corresponding modular theory is that by Schur. While the developments of Frobenius relate to the representation of a given finite group as a non-modular linear group the work of Dickson employs a modulus in such a representation.

UNIVERSITY OF ILLINOIS,
July, 1907.

THE DRESDEN MEETING OF THE DEUTSCHE MATHEMATIKER-VEREINIGUNG.

THE 1907 meeting of the Deutsche Mathematiker-Vereinigung was held in Dresden, September 15–21, in conjunction with the 79th convention of the Naturforscher und Aerzte. The meeting took place in room 80 of the Technische Hochschule. In commemoration of the 200th anniversary of Euler's birth a considerable number of the papers were devoted to an exposition of his services to science. The following papers were read:

1. K. ROHN, Leipzig: "Algebraic space curves" (report).
2. F. KLEIN, Göttingen: "Concerning the connection between the so-called theorem of oscillation of differential equations and the fundamental theorem of automorphic functions."

* Burnside, *Proc. London Math. Society*, vol. 3 (1905), p. 435.
† BULLETIN, vol. 7 (1900), p. 121.
‡ Loewy, *Mathematische Annalen*, vol. 64 (1907), p. 264.
§ Dickson, *Transactions Amer. Math. Society*, vol. 8 (1907), p. 389.

3. G. LANDSBERG, Kiel: "Theory of curvature and calculus of variations."
4. A. v. BRILL, Tübingen: "Introduction to the Euler commemoration."
5. L. SCHLESINGER, Klausenburg: "On a problem of diophantine analysis, as considered by Fermat, Euler, Jacobi, and Poincaré."
6. A. PRINGSHEIM, Munich: "Concerning Euler's transformation of series."
7. E. BRAUER, Karlsruhe: "Euler's theory of the turbine."
8. F. S. ARCHENHOLD, Treptow: "Some letters of Euler."
9. R. GANS, Tübingen: "Euler as physicist."
10. H. E. TIMERDING, Strassburg: "Euler's investigations in nautical mechanics."
11. W. HORT, Gross Lichterfelde: "Euler's significance in technical science."
12. E. HOPPE, Hamburg: "Euler's place in the theory of optics."
13. L. SCHLESINGER, Klausenburg: "Development of the analytic theory of linear differential equations since 1863" (report).
14. A. SCHOENFLIES, Königsberg: "On the so-called Richard paradox in the theory of point sets."
15. F. HAUSDORFF, Leipzig: "On dense types of order."
16. H. WIENER, Darmstadt: "Geometric theory of invariants of binary forms."
17. V. Varicak, Ogram: "Contributions to non-euclidean geometry."

1. Professor Rohn's report will soon appear in the Encyklopädie der mathematischen Wissenschaften, III, C. 8.

4. In a short paper Professor v. Brill called attention to Euler's birthplace, briefly sketched his life, expressed the belief that Germany's celebration in Euler's honor would find comprehending appreciation throughout the civilized world, and discussed its significance for the history of science.

7. Professor Brauer's paper will appear in the *Jahresbericht*.

9. Professor Gans gave the following historical summary: Euler published important memoirs in nearly all fields of physics. Above all, he created analytic mechanics, which, in

contrast with the earlier geometric method, supplies an ever-ready recipe for new as well as old problems. Before Euler's time even geniuses contented themselves with easy problems; since then, anyone can calculate by rule the path of a given mass-point under the action of determinate forces. Applying his theory to astronomy, Euler calculated by the method of variation of the constants, the perturbative effect of celestial bodies upon the orbit of a planet. He rendered assistance to the science of artillery by his approximative treatment of the path of a projectile; to nautical science by his theory of the ebb and flow of tides and of the building and sailing of ships. He investigated the bending and elastic swing of beams and calculated the safety load of a column, thus rendering service to technical science. He studied the theory of sound and investigated the physical basis of musical consonance and dissonance. The results of his work on optical instruments, which fill three thick volumes, have contributed much to the technique of microscope and telescope making, and, above all, have resolved the physics of catoptrics to that of dioptrics. His discovery of the achromatic telescope deserves especial mention, since it was at Euler's instigation that Dollond made his glass. Euler's ether theory is of great importance for theoretical physics and natural philosophy. He assumes the universe filled with a fine substance to whose vibrations the phenomena of light are due and whose currents are the ultimate cause of gravitation, electricity, and magnetism. It is not to be wondered at that this ether theory seems imperfect from our present standpoint, for in Euler's time Coulomb's law and the connection between light, electricity, and magnetism were unknown. Besides physical proofs Euler sometimes used theologic-teleological proofs, although his mathematical-analytic interest is frequently in the foreground. But in cases of application to astronomy or technical science no method is too uninteresting for him. We admire in him the powerful physical analyst, who studied the details, and the great natural philosopher, who sought the unities of natural phenomena.

10. Professor Timerding discussed those works of Euler which treat of naval mechanics. He indicated the principal points of view from which these works, especially the Scientia navalis, 1749, and the Théorie complète de la construction et de la manoeuvre des vaisseaux 1773, may be judged. Two things must be con-

sidered — personal relation to Euler's scientific temper and to his other investigations, and technical relation to other investigations concerning naval mechanics and to the development of rational naval construction. One should emphasize, for example, the relation to Euler's discovery of the laws of motion of rigid bodies and, on the other hand, the impulse to a critical investigation of the strain of the ship by reason of the coming into play of different forces. It should also be mentioned that Euler knew all three kinds of practicable propellers and had calculated their efficiency. The author emphasized the significance of Euler's systematic development for the subsequent progress of the theory of the ship.

No. 11 is to be published at once in the *Physikalische Zeitschrift.*

12. Professor Hoppe drew attention to the fact that Euler was the first to treat the vibrations of light analytically and to deduce the equation of the curve of vibration as dependent upon elasticity and density. Euler distinguished simple rays of homogeneous wave length from those of white light, calling them elementary rays generating elementary colors. He deduced the law of refraction analytically and explained that the rays of greater wave length must suffer the least deviation. He also investigated non-transparent bodies with regard to their behavior toward light. He studied dispersion in the search for a corrective for chromatic aberration, which Newton had declared unattainable. For this purpose he found the formula that the ratio of the natural logarithms of the indices of refraction must remain constant for different colors. If the ratio is α then the dispersion is given by $dn = (1 - \alpha)(1 + \alpha)^{-1} nln$. It was this investigation that induced Dollond to construct his achromatic lenses. Euler calculated a large number of composite telescopes and microscopes, thereby increasing the possibilities in the construction of these instruments. The results of his investigation with the composite ocular are of special importance. Euler treated atmospheric refraction of light analytically both for celestial and terrestrial objects, finding for the latter an expression that is fundamental in leveling. With regard to photometry Euler distinguished between intensity of light and of illumination and discovered the dependence of the latter upon the inclination of the surface and the decrease of the

former with the distance. He also called attention to the optical methods for eliminating the third dimension. Euler was thus the only physicist of the eighteenth century who advanced the undulatory theory.

14. Professor Schoenflies exposed the fallacy in the reasoning of Richard and showed that a parallel line of argument could be made to invalidate the results of all human thought.

15. Denoting by *series* (Reihen) sets of the type of Cantor's "Zahlclassen," or of the inverse type, one can assign a definite character to each element and to each gap of a dense set, according to the series which the element, or the gap, bounds to right or left. Upon this Professor Hausdorff based a classification of dense sets and an existence proof for the a priori obvious kinds. In the first case, in which the dense set contains series of potency one, there are two different species, represented by the linear continuum and by the set of rational numbers. In the case of series of potency two, there exist 210; of potency three, 243376 different species.

17. Professor Varicak's paper is in abstract as follows. Erect two normals to the x-axis at points $N_1(n_1, 0)$, $N_2(n_2, 0)$. There will be four straight lines gi parallel to these normals, uniquely determined by the intercepts n_1, n_2 and by the sense in which the gi shall be parallel. Making use of Lobachevsky's coordinates and assuming $\epsilon_1(\epsilon_2) = +1$ when the first (second) end of gi lies in the positive half plane, but equal to -1 in the contrary case, we can write the equation of these four parallels as follows:

$$e^x + \epsilon_1\epsilon_2 e^{n_1+n_2}e^{-x} - (\epsilon_1 e^{n_1} + \epsilon_2 e^{n_2})\,\mathrm{sh}\,y = 0.$$

This is the general equation of the straight line which can often be used to advantage. In the second part of his paper the author characterized some movements of the Lobachevsky plane by means of infinitesimal transformations. Translations along the "Abstandslinie," the normal and the parallel to the x-axis respectively are given by

$$Uf \equiv \frac{1}{\mathrm{ch}\,b}\cdot\frac{\partial f}{\partial x}, \quad Uf \equiv \frac{\partial f}{\partial y}, \quad Uf \equiv (1 - e^{-2x}\mathrm{sh}^2 b)\frac{\partial f}{\partial x} - \mathrm{sh}\,y\,\frac{\partial f}{\partial y}.$$

Rotations about the positive end of the x-axis and about the origin are given respectively by

$$Uf \equiv \frac{\sqrt{e^{2(x-b)}-1}}{e^{2(x-b)}} \cdot \frac{\partial f}{\partial x} + \frac{1}{\operatorname{ch} y} \cdot \frac{\partial f}{\partial y},$$

$$Uf \equiv - \frac{\operatorname{sh} y \operatorname{ch} x}{\operatorname{sh} p} \cdot \frac{\partial f}{\partial x} + \frac{\operatorname{sh} x}{\operatorname{sh} p} \cdot \frac{\partial f}{\partial y}.$$

Invariant pencils of curves can be determined easily by Lie's method.

At the business meeting of the Vereinigung a committee, consisting of Professors Pringsheim, Stäckel, and Krazer, was appointed to consider ways and means looking toward the publication of Euler's works. It was hoped that a report of progress could be made at the meeting in Rome next April. Professor Klein was elected to succeed Professor Pringsheim in the Vorstand and will be chairman for the current year. Professor Krazer was continued as Secretary.

During their stay in Dresden the visiting scientists did not lack for entertainment. The evenings were socially filled as follows: Sunday, September 15th, there was an informal initial gathering in the main hall of the permanent exposition building for the purpose of renewing old acquaintanceships and forming new ones. On Monday evening the Belvedere was elaborately decorated with colored lights in honor of the visitors. By reason of inclement weather the open air concert had to be abandoned, however, in favor of indoor entertainment. Tuesday evening the Naturforscher were the guests of the King of Saxony at a superb presentation of La Bohême at the royal opera house. Wednesday evening the visitors banqueted themselves, Thursday evening they were received at supper by the mayor of Dresden, in the main hall of the exposition building. During the entire week the city of Dresden, with its wealth of attractions, was hospitably open to the visitors, while abundant opportunity was offered to those who desired to make excursions, at reduced rates, to neighboring points of interest.

C. A. NOBLE.