THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE twelfth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of California on Saturday, September 28, 1907. The following members of the Society were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Mr. A. J. Champreux, Professor R. L. Green, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Mr. Joseph Lipke, Dr. J. H. McDonald, Professor W. A. Manning, Dr. T. M. Putnam, Professor Irving Stringham, Mr. J. D. Suter.

The following officers were elected for the ensuing year: Professor L. M. Hoskins, chairman; Professor W. A. Manning, secretary; Professor D. N. Lehmer, Dr. J. H. McDonald, Professor W. A. Manning, program committee. The next two meetings of the Section are to be held at Stanford University, February 29, 1908, and at the University of California, September 26, 1908.

The following papers were read at this meeting:

(1) Dr. J. H. MCDONALD: "On Minkowski's diagonal continued fraction."

(2) Professor H. F. BLICHFELDT: "Concerning the geodesics connecting five points."

(3) Professor D. N. LEHMER: "A report on Kulik's manuscript tables of factors."

(4) Professor D. N. LEHMER: "Extension of certain theorems in the theory of quadratic residues."

(5) Professor C. A. NOBLE: "Singular points of a simple kind of a differential equation of the second order."

(6) Professor G. A. MILLER: "Groups in which the subgroup which involves all the substitutions omitting a given letter is regular."

(7) Professor R. E. ALLARDICE: "Note on the cyclide of Dupin."

(8) Dr. J. H. MCDONALD: "On the condition that two circles may have a simultaneously in- and circumscribed quadrilateral."

(9) Professor L. E. DICKSON: "On quadratic forms in a general field."

The papers of Professors Noble, Miller and Dickson were read by title. Abstracts of the papers are given below in order as numbered in the list above.

1. Minkowski has drawn attention to the properties of a type of continued fraction development of a quantity x which is such that if p/q is a convergent, $|p - xq| < 1/2q$, and conversely if $|p - xq| < 1/2q$, p/q is a convergent. Minkowski's method is the use of a geometric representation. It is the object of Dr. McDonald's communication to show how to convert the usual development for x by a simple transformation into this diagonal continued fraction.

2. Professor Blichfeldt proved that a surface of revolution is the only surface possessing the following property: Five arbitrary points on the surface being selected, the ten geodesic distances connecting these points satisfy a relation independent of the coordinates of the five points.

3. Assisted by the Carnegie Institution of Washington, Professor Lehmer has made a careful comparison of his own tables with those previously existing. The first, second and third millions were compared, entry for entry, with Burckhardt's, the fourth, fifth and sixth with Glaisher's, the seventh, eighth and ninth with Dase's. The comparison will be made a second time and a list of errors published.

For the tenth million, Professor Lehmer hoped to avail himself of the manuscript deposited by Rosenberg's widow with the Berlin Academy. Unfortunately, these manuscripts seem to have disappeared. The manuscript tables of Professor Kulik, left in charge of the Vienna Academy, were however available, and with the generous permission of the heirs of Professor Kulik were obtained for examination and comparison.

The Kulik tables are remarkable for their extraordinary extent — to 100,330,201, according to the title page of the first of the six volumes (only one of the volumes was obtained for examination), and for the ingenious table of abbreviations. Primes up to 163 are represented by a single character: $a = 11$, $b = 13$, $c = 17$, etc., to $z = 109$. Then the digits are used (except 7, which was printed into the table as usual): $1 = 113$, $2 = 127$, $\cdots$, $9 = 157$, $0 = 163$. Zero and the letter o are distinguished in the manuscript by a stroke through the former. From 167 on, the primes are given by double symbols: $aa = 167$, $ba = 173$, etc. By using for the second letter all the letters of the Roman alphabet (j excepted) and all the letters of the German alphabet up to o, the primes are covered as far as 8057.

The arrangement of the table is the same as Burckhardt's. Each page of the manuscript contains 77 columns. The "sieve method" of making the entries was used for primes as high as 1009, after that the "multiple method" was used.

Something like two hundred errors were found in the manuscript for the tenth million. The exact number will be given after the second comparison.

4. Defining a quadratic residue as any number that is congruent to a square, modulo m, and leaving out the usual restriction that the number shall be prime to the modulus, Professor Lehmer obtains in his second paper formulas for the number of distinct residues of this sort.

It is a known theorem that for a prime of the form $4n \pm 1$ there will be found exactly n pairs of quadratic residues that differ by unity. This theorem was also extended to residues as defined above.

5. Professor Noble's paper will appear in full in the BULLETIN.

6. As the groups in which the regular subgroup G_1 is of degree $n - 1$, n being the degree of the group G, have received considerable attention, Professor Miller considers mainly those groups in which the degree of G_1 is $n - \alpha$ $(\alpha > 1)$. Some of the main results may be stated as follows: G contains n/α systems of imprimitivity which it permutes either according to the group of order 2 or according to a multiply transitive group. When G_1 is also abelian, G contains exactly α^2 substitutions which transform each of these systems into itself, and it contains an additional invariant subgroup of index $n/\alpha - 1$. The latter of these two invariant subgroups includes the former whenever $n/\alpha > 2$. In the special case when $n/\alpha = 2$ this subgroup reduces to identity. To arrive at these results frequent use was made of the following theorems: If a regular group H of degree n is transformed into itself by a substitution s of degree $n - \alpha$ in the same letters, then s is commutative with exactly α of the substitutions of H whenever $\alpha > 0$. When $\alpha = 0$,

s must be commutative with at least one of the substitutions of H besides identity. If a transitive group of degree n is transformed into itself by any substitution in the same letters and if the degree of this substitution is not $n-1$, then it must be commutative with at least one of the substitutions of the transitive group besides identity.

7. The object of Professor Allardice's note on the cyclide of Dupin was to show that, by means of a transformation originally due to Laguerre (see Darboux, Théorie des surfaces, volume 1, page 253), a circle may be transformed into this cyclide; and that the principal properties of the surface may be obtained geometrically by means of the transformation.

8. The relation between the radii and distance between centers giving the condition that two circles may have a simultaneously in- and circumscribed quadrilateral was obtained by various mathematicians (Fuss, Steiner, Jacobi, Cayley) in a form limited to a special case. The complete formulas are found by Dr. McDonald, incidentally giving the interpretation of certain results of the theory of elliptic functions.

9. Professor Dickson's paper appears in full in the present number of the BULLETIN.

W. A. MANNING,
Secretary of the Section.

ON QUADRATIC FORMS IN A GENERAL FIELD.

BY PROFESSOR L. E. DICKSON.

(Read before the San Francisco Section of the American Mathematical Society, September 28, 1907.)

1. WE investigate the equivalence, under linear transformation in a general field F, of two quadratic forms *

$$q \equiv \sum_{i=1}^{n} a_i x_i^2, \quad Q \equiv \sum_{i=1}^{n} \alpha_i X_i^2 \quad (a_i \neq 0,\ \alpha_i \neq 0).$$

An obvious necessary condition is that α_1 shall be representable by q, viz., that there shall exist elements b_i in F such that

$$\alpha_1 = \sum_{i=1}^{n} a_i b_i^2.$$

* Within any field F, not having modulus 2, any quadratic form of non-vanishing determinant is equivalent to one of type q.