classification, so essential if we are not to grope blindly. The following theorem gives the main results: *

A necessary and sufficient condition that a function be the limit of continuous functions is that it be pointwise discontinuous on every perfect assemblage.

Attention should also be called to the concept of semi-continuity, introduced in the proof of this theorem. The limit of the maximum values taken on by a limited function f in intervals (C, D) containing the point A, as (C, D) approaches zero, is called the *maximum* of f at A. If this equals the functional value at the point A, f is said to possess *upper semi-continuity* at A.

The book closes with a general classification of functions. Continuous functions constitute class 0; limits of continuous functions, not themselves continuous, form class 1; in general, limits of functions of class n, not belonging to class n, form class $n + 1$. No attempt is made to show that there exist functions in each class.†

In accordance with the general plan of the series, no advanced mathematical knowledge on the part of the reader is presupposed; this has fortunately compelled the author to give us a very clear and elegant treatment of certain portions of the theories of assemblages and transfinite numbers.

W. D. A. WESTFALL.

Differential Equations. By D. F. CAMPBELL, Armour Institute of Technology. New York, The Macmillan Company, 1906. 96 pp.

THIS is a text-book intended to give a student of engineering a short, practical course in solving differential equations. The author gives only the most common, straightforward methods and omits integrating factors, singular solutions, geometric interpretations, etc.

The first chapter recalls some theorems of algebra and calculus and gives the derivation of a differential equation from its primitive. The use of j instead of i to represent $\sqrt{-1}$ can hardly be considered an improvement. The second chapter deals with changes of the variable (which are practically never used in the rest of the book). It might well have given

* The proof has been simplified by H. Lebesgue, Leçons sur les fonctions de variables réelles par É. Borel, note 2.

† Lebesgue has proved that functions exist in each class, *Journal de Mathématiques*, 1905.

place to a short explanation of the geometric interpretation of differential equations. In chapter 3, equations of the first order and first degree are discussed. Only the more important types are treated — those which are linear, or can be made so; those which are homogeneous in x and y, or can be made so; those in which the variables are separable; and those which are exact.

In the chapter on linear equations with constant coefficients, the usual practice is followed of taking a value of y and proving that it satisfies the equation. In the case of a repeated root of the auxiliary equation, why not follow the same plan? Put $y = xe^{m_1x}$ and show that the equation is satisfied if m_1 is a multiple root. When the $D - a$ operator is introduced, the author goes more slowly than most writers and spends more time than usual in proving the fundamental properties. This should be an improvement, from a pedagogic standpoint. A number of practical examples are added at the end of this chapter. A solitary use for chapter 2 is found in the type of equation so unfortunately called "homogeneous linear." These, together with equations lacking x or y explicitly, are all the equations of order higher than the first which are dealt with. A short chapter is inserted dealing with integrable equations containing more than two variables, and the book closes with a chapter on partial differential equations. No attention is given to systems of equations save that two sample problems are worked; only six examples are given to be solved, so that the last chapter hardly justifies its presence.

Throughout the book the exercises are collected at the end of the chapters, but are arranged in classes, thus giving the student little practice in detecting the type to which any particular equation belongs. More might have been added in a number of places.

A number of misprints have been allowed to creep in. In § 12, c^a should be ce^a; in § 57, $z = 0$ should be $u = c$; at the end of § 65, "linear" should be omitted; the "familiar theorem of algebra" in example 2, § 69, needs amending. On page 33, sin xy, cos xy, etc., are used when it would be better to put y sin x, y cos x, etc.

C. R. MacInnes.