

congruence of lines is reflected or refracted on any surface into a normal congruence. It is shown that each sphere of the congruence touches the focal surface in two points. The congruence of lines formed by joining these points is discussed and some very pretty relations between the surface of centers of the spheres and their envelope are derived therefrom. The chapter closes with a short account of the cyclical systems of Ribaucour and Weingarten surfaces.

An excellent set of exercises is given to accompany each chapter.

C. L. E. MOORE.

*Nichteuklidische Geometrie.* Von HEINRICH LIEBMAN. Leipzig, G. J. Göschen (Sammlung Schubert, XLIX), 1905. 12mo. viii + 248 pp.

IN this volume of the Schubert collection, Professor Liebmann has succeeded in presenting an introduction to non-euclidean geometry that is brief, readable, and well-balanced. Its brevity will recommend it to the student whose interest in the subject has been aroused by the numerous references in literature, but whose time and maturity are scarcely sufficient for a study of the many longer and more difficult works. It might well appeal also to a teacher of elementary geometry. The recent literature on non-euclidean geometry naturally falls into two classes: the one dealing with the lives and writings of Lobachevsky, Bolyai, and Gauss; and the other consisting of systematic developments of particular phases of the subject. In this book there is a happy combination of the two methods, giving a broad outlook, and yet not sacrificing the unity.

The first chapter contains an interesting account of the parallel axiom and of the attempts at its proof, considered from an historical point of view. The next five chapters, comprising three fourths of the book by pages, are devoted to hyperbolic geometry, beginning with a very simple account of its picturing by means of circles in the euclidean plane. One regrets that references are not given here to some, at least, of the articles that have appeared during the last twenty years on this picturing. With this one exception, the many exact references to a comparatively wide range of literature form one of the most attractive features of the book. There are other chapters on hyperbolic geometry, dealing with the synthetic and the analytic geometry and the trigonometry in the hyperbolic plane. After

this more detailed account, the spherical-elliptical geometry is disposed of in one short chapter. The last chapter is on non-euclidean mechanics; and the book is concluded by a few pages in which attention is directed to some of the discussions about the nature of actual space.

E. B. COWLEY.

*A Brief Introduction to the Infinitesimal Calculus.* Designed especially to aid in reading mathematical economics and statistics. By IRVING FISHER, Ph.D., Professor of political economy in Yale University. Second edition. New York and London, The Macmillan Company, 1906. 12mo. xiii + 84 pp. Price, 75 cents.

THIS book gives an excellent bird's-eye view of the differential calculus, and indeed of the integral calculus. It is written with remarkable clearness, the illustrations from geometry, physics and economics being well chosen and well placed. In this, the second edition, the notion of the "little zero" is not used. Its use in the first edition was criticized by Professor Fiske in his review of the book in the BULLETIN, February, 1898, page 238.

Though small, the book is very comprehensive. If it were to be enlarged, the first addition would perhaps be an article on the mean value theorem, of which article 69 is suggestive, and a page or two on integration as summation, in place of the two short articles 76, 87. Some footnotes, such as the one inserted in the German edition (Teubner, 1904) for article 35, would add to the logical completeness of the proofs, and a few slight changes might be made in the introductory chapter.

The book contains about 200 well selected problems, and is an admirable text-book. It supplies the need, felt by some, of a text-book for those who wish to become familiar, in a short time, with the fundamental conceptions of the calculus.

EDWARD L. DODD.

*Leçons sur les Fonctions Discontinues.* Par RENÉ BAIRE. Rédigées par A. DENJOY. Paris, Gauthier-Villars, 1905. 8vo. viii + 127 pp.

IN these Leçons the Borel series of monographs has given us a work of fundamental importance in a too long neglected field. The interest in discontinuous functions is happily increasing, and finds in this little book a basis for attack and for