(page 28) he speaks of "the quantity $\sqrt[b]{a}$, of which we have above shown the existence." We are told further (page 28) that in case of continuous magnitude the presence of quantities other than the rational is necessarily imposed by the assumption of the infinite divisibility of such magnitude, "for otherwise the division into parts would be necessarily limited." This sounds very clear and simple; and yet we are troubled by the fact that infinite divisibility in M. Dassen's sense seems to us to imply not necessarily a continuum, but only an aggregate everywhere dense in itself, for the representation of which the rational numbers suffice.

Enough has probably been said to indicate the character of M. Dassen's philosophy and the mathematical knowledge on which it is based, so that we may refer anyone interested in its further development to the remaining two chapters of the book itself, which have to do with directed quantities in the plane and in space respectively. In an appended note the author pays his respects to the work of Tannery and Kronecker regarding the founding of analysis on the concept of the positive integer alone, and pronounces it quite useless and a mere jugglery of symbols, "at which one is justly shocked." From what precedes, the fact that he has entirely missed the real object of such work is not surprising.

Before closing, we would, however, refer to a feature of the work which is of considerable interest. The author has scattered through the text a very large number of historical data. These are quite independent of his philosophy and seem to be drawn from reliable and often not easily accessible sources.

J. W. YOUNG.

A Treatise on Differential Equations. By ANDREW RUSSELL FORSYTH. Third Edition. Macmillan and Co., 1903.

IT is not an easy matter to review a book which, like the present one, has been before the public so many years, the first edition having appeared in 1885 and the second in 1888. That this treatise has many virtues has been quite conclusively shown by its success. In fact in English-speaking countries the domain of differential equations has, since 1885, been synonymous with the name of Mr. Forsyth, at least in the minds of that great category of students whose knowledge comes from text-books only.

As to the arrangement and the general contents of the book it would be useless to make any comment; it is all so well known. The book is a most useful one. It is very much to be regretted however that, in a considerable number of instances, the statements are given in a loose and misleading manner. On page 4 for example we find the following remark: "In the example considered, the equation giving dy/dx had only a single root; when it is of the form

$$\left(\frac{dy}{dx}\right)^2 + P\frac{dy}{dx} + Q = 0,$$

then the integral equation will be of the form

$$A^2 + AP' + Q' = 0,$$

where A is an arbitrary constant, etc."

Clearly, this statement without any restriction as to the domain of rationality is without value, and in a book intended for beginners it is misleading. Similarly the statement of the problem of integrating a differential equation given on page 6 is destined to perpetuate an erroneous conception. The author says in italics: "In fact every differential equation is considered as solved, when the value of the dependent variable is expressed as a function of the independent variable by means either of known functions or of integrals, whether the integration in the latter can or cannot be expressed in terms of functions already known." Again on page 51 he says: "Cases occur in which reduction to quadratures is not possible, that is to say, *the equation cannot be solved analytically.*" (The italics are mine.) Of course the author knows that analytic integration of equations not reducible to quadratures *is* possible; his great work on the "Theory of differential equations," which was reviewed in this BULLETIN some years ago, shows him to be thoroughly familiar with the modern ideas. But no one would imagine this to be the case from reading his "Treatise." Of course it is justifiable to divide the theory of differential equations into two parts, one accessible by elementary means and the other depending upon the most advanced ideas of modern analysis. But why make statements in the elementary portion which must be contradicted later on, when a few words would suffice to explain the situation?

Lie's theory of continuous groups plays such an important part in the elementary problems of the theory of differential equations, and has proved to be such a powerful weapon in the hands of competent mathematicians, that a work on differential equations, of even the most modest scope, appears decidedly incomplete without some account of it. It is to be regretted that Mr. Forsyth has not introduced this theory into his treatise. The introduction of a brief account of Runge's method for numerical integration is a very valuable addition to the third edition. The treatment of the Riccati equation has been modified. The theorem that the cross ratio of any four solutions is constant is demonstrated but not explicitly enunciated, which is much to be regretted. From the point of view of the geometric applications, this is the most important property of the equations of the Riccati type. The theory of total differential equations has been discussed more fully than before, and the treatment on the whole is lucid. The same may be said of the modified treatment adopted by the author for the linear partial differential equations of the first order. A very valuable feature of the book is the list of examples.

E. J. WILCZYNSKI.

Introduction to Projective Geometry and Its Applications. By ARNOLD EMCH. New York, John Wiley & Sons, 1905. vii + 267 pp.

To some persons the term projective geometry has come to stand only for that pure science of non-metric relations in space which was founded by von Staudt. To others the original significance of the word, implying an actual projection of one metrical space upon another, still remains essential. The author of this book belongs to the latter class. He starts from metric and descriptive geometry. In the development of the matter treated in the text there is no trace of any kind of purism. Analytic and synthetic methods are everywhere used impartially. The result is a book which will certainly appeal to students of engineering and others who desire to use projective geometry in practical work.

Although there is to be found, especially in the later chapters, much which should interest students of pure mathematics, there are a number of defects which cannot but detract from their enjoyment of the work. These seem to be in a great measure a matter of style. Thus on page 19 we find in italics :