Cardan's symbols, and Scheubel's, and the interesting use of $L2$ for $\surd 2$ and $2\ L$ for $2\ x$, by Ramus, and various other curious and common forms, are unmentioned. Indeed, the work offers but little help in reading the older authors, and the chapter must accordingly be considered disappointing.

This topic is followed by a review of the general theory of algebraic equations, trigonometry in relation to algebra, numerical computation before the invention of logarithms, the invention of the latter, the number theory before and during the time of Fermat, the theory of combinations and probability, and the projective and analytic geometry of the seventeenth century.

The third and most important section of the work, and that for which it will probably be chiefly consulted, relates to the invention of the calculus. Professor Zeuthen rightly sets forth the important bearing of mechanics upon the early steps in this theory. He shows that applied mathematics demand some form of integration, and that in the efforts of Kepler, Cavalieri, Fermat, Pascal, Wallis, and others, must be sought the first evidences of the modern integral calculus. He then shows that the study of tangents before Newton's time demanded an approach to differentiation, as seen in the labors of Torricelli, Roberval, Descartes, Hudde, Fermat, Huygens, and de Sluse, and, he might well have added, Barrow.

The work closes with a critical study of the respective merits of Newton and Leibnitz in the invention of the calculus, adding nothing that is new to the long drawn out controversy, but agreeing with the general view of the present day as to the priority of the former and the independent work of the latter.

In spite of the incompleteness of various portions of the work, and in spite of the absence of all bibliographical aids to the student, Professor Zeuthen's work will justly rank as a worthy contribution to the history of the two most interesting centuries in the historical development of mathematics.

DAVID EUGENE SMITH.

Introduction à la Théorie des Fonctions d'une Variable. Tome I. By JULES TANNERY. Paris, Hermann, 1904. ix + 419 pp.

THE excellence of Tannery's book of 1886 as an introduction to the theory of functions of a real variable is so generally conceded that it seems sufficient in a notice of the revised edition to point out clearly that the new work in two large volumes is virtually a new treatise, constructed on broader and more ambitious lines, but still possessing the simplicity, clearness, and inimitable style of Tannery's earlier writings. The new work will doubtless at once supersede the earlier.

The present first volume treats in order the subjects irrational numbers, arithmetical continued fractions, infinite assemblages (ensembles), their upper and lower bounds and their points of condensation, infinite sequences, limits, denumerable aggregates, infinite series and products (covering a hundred pages of the text), general functions, polynomials, rational functions, uniform convergence, exponential and trigonometric functions, calculus of derivatives, study of functions by means of their derivatives, Weierstrass's example of a continuous function not possessing a derivative. It will be observed that the present volume contains practically all the subject matter of the first edition, except the subject of definite integrals. The latter, together with the theory of functions of a complex variable, will be treated in the second volume.

There is a marked contrast between the methods of presenting the fundamental concepts in the present and earlier books. The theory of infinite aggregates is now in the foreground from the outset. It is only after the notions of bounds and points of condensation have been gained that the reader is introduced to sequences and limits. As relating directly to the theory of aggregates, irrational numbers are now considered exclusively from the point of view of Dedekind. In the earlier book, this treatment entered parenthetically (pages 10–24) as a method alternative to that in which an irrational number is defined by a convergent sequence.

There have been writers who took pride in the fact that no geometric diagrams occurred in their books. With regard to the contrast in this respect between his earlier and present books, Tannery takes the public into his confidence in the following words: "Si, dans la première édition, je me suis abstenu de tout langage et de toute figure géométriques, c'est que, dans les limites que je m'étais imposées, l'emploi de ce langage ou de ces figures n'aurait guère simplifié les raissonnements, c'est aussi à cause d'une certaine timidité; j'avais peur que le lecteur ne fût pas bien persuadé, s'il voyait une figure, que cette figure n'était qu'une aide et ne cachait point quelque trou, impossible à combler avec les seules ressources de la logique, et, peut-être aussi, avais-je besoin de fortifier en moi-même la conviction que je désirais imprimer dans son esprit.

Aujourd'hui, les craintes de cette sorte me semblent un peu puériles; j'ai d'ailleurs l' intention, pour ce qui concerne les éléments de la théorie des functions d'une variable complexe, tout en restant dans le domaine du nombre, de profiter des facilités qu'offre le langage géométrique."

While a studious avoidance of geometric language, lest geometric intuitions run rampant, does not seem necessary in attaining purity of logic in analysis, the user of such phraseology must be able to say with Sophus Lie, "Ich kenne meine Leute."

The pages seem to be as free from errata as they are attractive in appearance. In 2° of page 66, M should read M'. The steps in Tannery's arguments are nearly always very obvious. If (E) is an assemblage with an upper and lower bound, and (E') is its derived assemblage, it is stated in No. 83 that the assemblage $(E) + (E')$ is closed. This becomes evident to the reader after he has the theorem of No. 85 that (E') is closed if it is infinite.

It is interesting to note that the appearance of this valuable treatise was simultaneous with the announcement of M. Tannery's appointment to the professorship of calculus in the faculty of science at the University of Paris.

L. E. DICKSON.

ERRATA.

The following errata in the present and the preceding volume of the BULLETIN have come to the attention of the editors:

Volume 10.

On page 222 the number of papers read before the Society in 1903 should have been reported as 148.

Volume 11.

Page 213, line 22, *for* all *read* and.
Page 477, lines 20, 27, *for* σ *read* 0.