respectively and represents rotations about μ and λ in the reverse directions. The changes in the figure required by this reversal are obvious.

PACIFIC GROVE, CALIFORNIA,
February 23, 1905.

SHORTER NOTICES.

Introduction a la Géométrie Générale. By GEORGES LECHALAS. Paris, Gauthier-Villars. Pp. ix + 58.

THE point of view of M. Lechalas is different from that of most recent workers on the foundations of geometry. Ordinary usage at present applies the term mathematical science to any body of propositions deducible from a set of postulates. The term geometry is applied with various degrees of freedom to a large number of sciences, all however characterized by similar types of relation and modes of study. These abstract geometries are "represented concretely" in many different ways, as is illustrated for example, by the beautiful theorem of M. Barbarin :*

"Each of the three spaces, Euclidean, Lobachevskian, Riemannian, contains surfaces of constant curvature of which the geodesic lines have the metric properties of the straight lines of the three spaces."

To Lechalas, on the other hand, a geometry is a "form of externality." Under the title General Geometry he includes only the geometries of Euclid, Lobachevsky and Riemann (double elliptic or spherical geometry), thus excluding from consideration not only the current "bizarre geometries" but even the symmetric non-archimedean geometries of Hilbert and Veronese as well as the classical "single-elliptic" geometry. In no place do we find clear distinctions between metaphysical and mathematical questions, in a book where both are considered. On the contrary we find the following statement (page 16), which reads rather strangely in view of the vast number of different ways of representing an abstract science to the imagination :

* Quoted thus in a translation by G. B. Halsted of a report by P. Mansion on the non-euclidean researches of P. Barbarin, *Science*, n. s., vol. 20, No. 507 (September 16, 1904).

"Nous l'avons déjà dit, nous ne saurions accepter comme géométrie une analyse imaginaire, telle que celle qu'on obtient en remplaçant R par $R\sqrt{-1}$ dans les formules de la géométrie sphérique, car, là, la représentation par l'imagination n'est pas seulement impossible en fait, mais elle l'est absolument: cette analyse n'a plus aucun rapport avec une forme d'extériorité."

Curiously enough, the paragraph just quoted is followed by this: "Au contraire, la notion d'un espace à quatre dimensions repose sur des intuitions spatiales, et nous n'avons aucune raison de refuser d'admettre qu'elle pourrait répondre à des images, soit dans d'autres esprits, soit même dans les nôtres si notre sensibilité avait reçu une autre éducation."

The exposition of the usefulness of a four-dimensional point of view in studying a three-space is the best thing in Lechalas's book. He presents the distinctions between symmetry and congruence, etc., both for euclidean and spherical space, in a very elegant and vivid way, suggesting strongly the desirability in general of studying the space of a given type when situated in a space of higher number of dimensions of the same or another type. It is in this respect that Lechalas comes nearest to carrying out his avowed aim to present the subject in a style less "fragmentary" than that which its historical development has forced upon geometry.

His principal other means to this end is the introduction of the concept of curvature of a surface as independent of the space in which the surface is situated. This is done by means of the theorem of Gauss, that the integral curvature of a geodesic triangle is equal to the sum of the angles of the triangle diminished by two right angles. A geodesic line is, however, neither clearly defined nor clearly recognized as undefined. If the geodesic is accepted as an undefined element, such for example as the straight line in Hilbert's geometry, then the actual procedure of Lechalas (cf. his Chapter III) is not very different from the usual one of putting the assumptions about parallel lines in the form of statements about the angle-sum of triangles.

The book is intended primarily for readers "who have not yet formed a systematic conception of the three geometries." Its usefulness for this purpose is destroyed, in the opinion of the reviewer, by the polemical (though courteous) attitude adopted toward MM. Mansion and Barbarin. The controverted questions are such as hardly can arise if one takes the abstract point of view, and they certainly have very little interest for American readers.

OSWALD VEBLEN.

Elementare Algebra. Akademische Vorlesungen für Studierende. By Professor Dr. EUGEN NETTO. Leipzig, B. G. Teubner, 1904. viii + 200 pp.

THE book under review is written in the informal and somewhat detailed style of the lecture as distinguished from a treatise, is rather generously supplied with well-chosen figures, and contains a good index but no exercises for the student to solve. It is the outgrowth, as the author tells us in the preface, of a course of lectures (entitled Einleitung in die Algebra) which he gives during the summer semester of each year in the University of Giessen.

The purpose of the book, and also of the lectures on which it is based, is two-fold: for those students who are to continue their work in mathematics, it is designed to bridge over the gap which usually exists between the algebra of the fitting school and that of the university; and to the non-mathematical student it presents, in a somewhat popular form but from a broad viewpoint, some of the more important problems and methods of algebra, a knowledge of which should have a place in a liberal education.

Equations of the first four degrees determine the main divisions of the book, and around these cluster a great variety of topics. Thus there are eight chapters with the following titles: I Equations of the first degree; II Pure quadratic equations; III General quadratic equations; IV Permutations and combinations; V Determinants. Linear equations; VI Binomial equations; VII Cubic equations; VIII Biquadratic equations. And among the many topics discussed under these headings may be mentioned: in Chapter I determinants of the second order, arithmetical progression, continued fractions, and indeterminate equations; in Chapters II and III Newton's method of approximation, periodic continued fractions, imaginary and complex numbers (including a rather full graphic treatment), probability that the roots of a random quadratic are real, simple and multiple valued functions and discriminants; with an equally richly varied selection for the other chapters, — Chapter V is particularly well done.