THE OPPORTUNITIES FOR MATHEMATICAL STUDY IN ITALY.

THE question where a young mathematician shall complete his studies is one of ever recurring interest. We are in the habit of referring to Emerson's $\Phi B K$ address as the "Intellectual declaration of independence of America" and we rejoice to feel that with those memorable words we threw off our yoke of bondage to the Old World. On further reflection, however, we admit that Pasteur's epigram may be inverted; that if a man of science has a fatherland, science itself knows none, and the wise student will always, like Mahomet, go to the mountain. He will complete his education in that country, and at that university where, for his special needs, the best subjects are best taught.

American students of mathematics, who intend to devote their lives to the subject, have generally in the past deemed it wise to spend more or less time at a European university, and though we may point with pride to the work that is being done to-day in our own graduate schools, it is safe to predict that for a long time to come, this migration will continue. Now to most Americans the words "study in Europe" and "study in Germany" are synonymous, and the reasons for this are not far to seek. The prestige of the German universities, their well deserved renown for free research and profound scholarship, and the scientific standing of those who teach there, have been, and will continue to be, the determining motives leading our students to the German Empire. And yet the whole truth has not been told when we have merely praised the German universities. Other universities have come forward in the last decades; Germany has no "corner" in science nor yet in the pursuit of scholarship, and to believe so is a sign of a sad, if all too common, intellectual provincialism. Moreover it is a highly debatable point whether our own American mathematical scholarship has not had a full measure of German influence, and whether the time is not come when we should show ourselves more ready to assimilate ideals from other lands. We should, I am sure, be no losers by having among us more teachers and students who had frequented the other European universities, nor

would such persons look upon themselves as having enjoyed only the second best in education. Yet at this point we are confronted with a difficulty, for, compared with Germany, the conditions of life and study elsewhere in Europe are imperfectly known and understood in America. Hence Professor Pierpont's article upon "The teaching of mathematics in France" * was most opportune, and in a similar spirit I wish to give some account of opportunities for mathematical study which an American will find in Italy. I will preface what I have to say with the remark that I have had the privilege of attending only one Italian university, Turin; but as all the universities are under the immediate control and supervision of the State, and the broad outline of the course of instruction is prescribed by law, the same for all, what I have to say should apply with but slight modification to every university in the peninsula.

The American students who attend a European university may be divided into two classes, those who are seeking a doctorate, and those who have come to study merely a particular subject, or to hear a particular professor. It is my wish to show that for the former class of students the Italian universities are, as at present constituted, ill adapted, but the latter class might frequent them to great advantage.

The prescribed mathematical course in the Italian universities presents a certain superficial resemblance to that which is followed at Oxford and Cambridge. The period of residence is four years, and that term is marked by two great series of examinations. The first examinations, which are divided between the first and second years, lead to the "licentiate" or licence to teach in the lower schools. The final examinations lead to the "laureate" and confer the title of "doctor." They constitute the test required of all who wish to teach in gymnasia. A student is not admitted as a candidate for the laureate until he has passed the licentiate.

The requirements for the licentiate are the same for those who wish to pursue mathematics, physics, or engineering. After presenting proof that he has received a good school education, the candidate is required to attend the following courses:

1. Experimental physics for two years.
2. Inorganic and organic chemistry.
3. Algebraic analysis.

* BULLETIN, 1899.

4. Infinitesimal analysis.
5. Mineralogy.
6. Analytic geometry.
7. Projective geometry with design.
8. Descriptive geometry with design.

As American graduate students are scarcely likely to follow these courses, we may pass over them with no further comment than to point out the particular emphasis laid on the general subject of geometry.

A candidate for the laureate is allowed a certain freedom in the choice of subjects he will follow. He is required to follow the course in rational mechanics, and beside that, four courses chosen from these six:

1. Theoretical geodesy.
2. Mathematical physics.
3. Higher analysis.
4. Higher geometry.
5. Higher mechanics.
6. Astronomy.

The last course (astromony) is not given in all universities.

In some of the subjects, besides the lectures, there are recitations with problems, like the "travaux prâtiques" at the Sorbonne, presided over by an assistant. The student generally offers himself for examination in at least three of his five courses at the end of his first year of candidacy; his second year is largely occupied with his dissertation, and the preparation of his minor theses.

It must be confessed that, to an American eye, this list of subjects is somewhat strange. It is not likely that many students will care to earn the title of "dottore" in mathematics without studying either higher analysis or higher geometry, but they are at liberty to do so. If the professor of higher analysis happens to be lecturing of the theory of forms, the student may go forth with only a rudimentary knowledge of differential equations; if the subject be elliptic integrals, he probably will have little more than a bowing acquaintance with invariants. Most surprising of all is the placing of theoretical geodesy on a plane with the other subjects.

The examinations are all oral, and cover only the material discussed in the course of lectures delivered that year. There are many grades of laureate, and the student who has read nothing but what was suggested in lecture is not likely to pass out with "full wishes and praise" as the highest grade is called, nor will the most favorable opportunities probably be waiting for him in future; still he has the highest degree which the university confers.

The dissertations, as is natural, present great variation in value and originality. The candidate is required to publish a full résumé of his results, but not the thesis itself; whereby a long suffering mathematical public is spared much, for the feebler dissertations never see the light. The better ones find ready acceptance by the mathematical journals, or are laid before learned societies, and need not fear comparison with the work of aspirants for the German Ph.D. The preparation of the minor theses is much less of an affair, and usually occupies the candidate but a few weeks. No great originality is expected; he is merely called upon ordinarily to make a comparative study of various expositions of the assigned topics, and master those which he prefers.

It will be seen, I think, that the Italian laureate will not attract many American students as a substitute for the Ph.D. Few would care to devote time to mastering, not the five subjects, with which they may be more or less familiar, but the presentation of those subjects which had been given at that university. Fewer still would care to begin their course by offering themselves for examination in the diversified subjects of the licentiate.

These difficulties disappear in the case of the student who comes for a shorter time, say a semester, or a year, to study a particular subject with a particular professor. The requirements for such studies are libraries, lectures, and personal help. Where are they best supplied? In England the lectures suffer from the fact that they are purely utilitarian, means to prepare students for the great examinations. In Paris the Bibliothèque nationale is difficult of access; the library of the Sorbonne is overcrowded, and open only a limited number of hours every day; the smaller libraries, like that at the Ecole Normale, are ill supplied with funds. Moreover, in Paris the personal element is notoriously lacking. This difficulty exists in some measure also at the larger German universities, owing to the larger number of those wishing to be helped; in the smaller ones, the choice of lectures is naturally somewhat restricted, nor is it easy to find in Germany any lectures exhibiting the charm and finish of those delivered by the great French teachers.

In the better Italian universities, these three desiderata seem to be very happily combined. In Turin there is an excellent mathematical library, opened twice daily, from which with the

professor's permission students may withdraw what books they will. The large national library is in the university building, and is open all day to whoever wishes to enter. Lastly, the privilege is sometimes granted of using the fine library of the Royal academy of science, a delightful place, which rejoices especially in possessing the proceedings of all imaginable learned societies. I doubt whether anything corresponding to this last collection could be found in those universities which are in the smaller cities; duplicates of the two former would certainly exist in any of the larger Italian universities.

The general plan of lectures, as outlined above, is determined by the State. At the same time such headings as "Higher analysis," "Higher geometry" are so comprehensive as to leave to the teacher the greatest discretion in the choice of material. Some fortunate professors give a new course each year, others run through a cycle including a greater or less number of subjects. Americans are sure to find lectures on subjects that will interest them, and they will have the French, rather than the German standard of clearness and elegance. They will also be struck by the eclecticism of the instructor, for Italian mathematicians read widely. I remember being impressed at the beginning of one course of lectures by the fact that the professor put down, as principal works of reference, books in four different languages, and remarked that those of his hearers who could not read English, French and German, must certainly make up the deficiency in the course of the year.

The pleasantest feature of the life of an advanced student in Italy will be the personal contact with that professor in whose subject he is specially interested. Seminars do not exist, nor have I seen traces of students' mathematical clubs, but the relation between teacher and pupil is most direct and most helpful. The professor expects to give help and advice to those students who are preparing dissertations upon his subject, and these he will cordially extend to any student pursuing a similar line of work.

The principal difficulties which an American encounters at an Italian university are of a formal sort. Should he wish merely to come to lectures as a guest of the professor, he will be more than welcome. Should he feel that he had no right to accept so much without enrolling himself and paying the usual fees, he will need some patience to fulfill all the formalities.

We ordinarily suppose that the English universities stand in a class by themselves in the matter of ill-contrived machinery, and that their regulations are the most hampering, but this is a mistake. True it is that an American who studies at Oxford must wear academic dress at the proper times and seasons, and must forego the dangerous habit of carrying weapons such as javelins, while at Cambridge he will not be allowed to play marbles on the steps of the Senate House. In larger matters the regulations are elastic and reasonable. In Italy, however, the statutes are planned for one type of student wishing one thing, and when a student presents himself who wants something else the process of adjustment is difficult and jarring.

There are three ways in which an American may enter an Italian university. To begin with, he may lay aside his dignity, and enter the first year. The difficulty here is that he must show a certificate of a good school education, with a written declaration that in America such a certificate would qualify for admission to a university. This certificate must be countersigned by an Italian consul in America. Secondly he may enter for one of the later years. Here the school certificate is to be replaced by documentary evidence that he has received instruction equivalent to that given in the years passed over in Italy. This also requires the consular signature. In addition, as the student has not migrated from another Italian university, he must begin by paying all the dues for those years which he has omitted. It is doubtful whether this method will ever be popular among Americans. The third way is to register as an "auditor." The formalities which must be fulfilled here, and which will be explained step by step, are the following:

1. Apply for registration on a prepared form and copy the application on a piece of stamped paper (to be bought at any tobacconist's), taking care not to run over into the margin.

2. Pay at the sub-treasury twelve lire (say $2.40) per course of lectures per semester.

3. Pay to the university bursar ten lire for "eventual damage" to the plant, and three lire fifty for a matriculation book.

4. Deposit the receipts for these sums at the proper office, together with two copies of the candidate's photograph, of the official size and shape. For these special arrangements are made with certain photographers.

5. Deposit a birth certificate countersigned either by an Ital-

ian consul in America or some personage recognized by the authorities. The student will be allowed to attend lectures from the beginning, and need not wait until he has completed all the requirements.

The birth certificate requirement always appears strange to an American. I was told that it was most important as a means of verifying the truth of my declaration concerning my age. I have never been able to find out what should happen in the case a candidate appeared declaring that such a thing was unprocurable. Probably this possibility has never occurred to any one. I remember hearing from an American who had been at the Ecole normale at Paris that there also they had demanded a birth certificate, and when he said that he had none, they were pained, not so much by his failure to produce the document, as by his obvious untruthfulness in saying that he did not have it. I am very sure that an American who could honestly say that a certificate of his birth was non-existent would not be refused registration on that account, though how the matter would be settled I do not know.

The further requirement of countersigning this precious document is, at first sight, exasperating. In my own case I heard not a word of it until I had received the certificate sent out at my request from America. I appealed for a dispensation, first to the Rector, and then through the American ambassador to the Minister of public instruction, but was refused. In the end I sent the certificate to Rome, where our embassy took charge of the formalities, and this might be the easiest way for Americans to follow in the future.

The Italian scholastic year is blessed with many holidays. The long vacation practically extends from the middle of June to the first of November. Christmas and Easter each provide ten days of vacation, while the carnival furnishes another week. Then there are six religious, and five national holidays. Lastly there are unannounced holidays. At periods of political excitement the students have a patriotic custom of marching about in bands, invading the lecture rooms, and calling on the professors for speeches. Should the doors be locked they will applaud enthusiastically thereon with hands and feet. As there is no discipline to deal with these cases, and it is not etiquette to introduce the police, the authorities frequently adopt the prudent course of closing the university until word arrives that the students are ready to return to ordinary academic ways.

Another picturesque custom prevails in some of the larger courses. If the professor be late in coming to lecture, the students indulge in a pandemonium of assorted noises till he arrives. This seems to discourage the adoption of the "Academisches Viertel." It is in ways like these that the students betray what is traditionally called a spirit of youth and gaiety suitable to their age (say eighteen to twenty-two) and in strong contrast to a certain, unmistakable intellectual maturity. This latter is striking. At the beginning of the present year I had the pleasure of hearing the truly remarkable opening address delivered by Professor Pio Foa. The subject was the necessary correlation in education of the speculative and experimental sciences, and from the first word the discussion was maintained at an intellectual level adapted to the most scholarly audience. The day was warm, the hall ill-ventilated, and that part of it reserved for students was without seats. There they stood, jammed together for an hour, giving perfect attention. At the close they applauded the speaker warmly, crowding enthusiastically about him as he left the hall.

About the social life of Italian students I know little. In Turin there is a large body called "The students association" which has some rooms, and a chapter of the international society "Corda Fratres," which publishes a magazine and rejoices in many officers. There is also a students' fencing club. Anything like the corporate life of an English college or a German "Verbindung" seems entirely lacking.

The cost of living in Italy is low according to American standards, although I should judge a little higher than what a student would pay in some parts of Germany. Furnished rooms are easy to find, as is also "pension de famille." In Turin the number of resident foreigners is almost a negligible quantity, compared to the crowds who go to other cities. Hence the tourist fares worse than elsewhere, but it is easier to enter into the life of the people, and the student should always try to associate with the native rather than the foreign population.

The question of language should not deter any one from coming to Italy to study, for if Italian is a bad language to speak easily, it is an easy language to speak badly. A student who should give a fair amount of time to studying any one of the many "methods" for learning the language during the summer vacation preceding his residence, and spend the last

month of that time, October, studying in Italy, will not be an accomplished Italian scholar, but he will have no trouble in understanding lectures, and in living comfortably in an Italian community.

There is, lastly, the question of the vacations. It is neither necessary nor suitable to dwell here upon the inestimable advantage to a student of spending his vacations in Italy, taking little journeys hither and thither. The serious student will choose his university by what it offers him in term time, not by what he can get out of term. Yet he who decides to come to Italy will realize that he will have the chance, with little trouble and small expense, to add to his special work a wider culture which no amount of mathematical study can give him, nor yet entirely destroy.

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VECTOR ANALYSIS.

Vectors and Rotors, with Applications. By O. HENRICI and G. TURNER. London, Edwin Arnold, 1903. xv + 204 pp.

Introduction to Quaternions. By KELLAND and TAIT. *Third Edition*, by C. G. KNOTT. London, Macmillan and Co., 1904. xvii + 208 pp.

Vektordifferentiation und Vektorintegration. Von V. FISCHER. Leipzig, J. A. Barth, 1904. iv + 82 pp.

THE first years of this century have seen an interest in vectors and related subjects, such as has never before been observed. This may be due somewhat to the activity and broad spirit of the International association for the promotion of the study of quaternions and allied systems of mathematics; but more probably it is caused by the increasing desire of physicists for a notation which will represent more briefly and more concretely the three dimensional quantities with which they are forced to deal. With this demand on their part the champions of the various systems come forward to press upon us the advantages of their particular system and especially the disadvantages of all others. Felix Klein is at the head of a commission to investigate the relative merits of different methods, and a number of articles in the recent