THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

A regular meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, October 31, 1903. About fifty persons attended the two sessions, including the following thirty-nine members of the Society:

Professor Joseph Bowden, Professor E. W. Brown, Professor F. N. Cole, Miss E. B. Cowley, Miss L. D. Cummings, Dr. W. S. Dennett, Dr. L. P. Eisenhart, Professor Achsah M. Ely, Dr. William Findlay, Professor T. S. Fiske, Dr. A. S. Gale, Dr. G. B. Germann, Dr. C. N. Haskins, Mr. L. L. Jackson, Professor Harold Jacoby, Mr. S. A. Joffe, Dr. Edward Kasner, Dr. O. D. Kellogg, Professor C. J. Keyser, Dr. G. H. Ling, Mr. L. L. Locke, Professor James Maclay, Dr. C. M. Mason, Professor W. H. Metzler, Professor W. F. Osgood, Professor E. D. Roe, Miss I. M. Schottenfels, Professor Charlotte A. Scott, Dr. Arthur Schultze, Mr. Burke Smith, Professor D. E. Smith, Mr. E. R. Smith, Professor P. F. Smith, Professor Virgil Snyder, Professor H. D. Thompson, Professor E. B. Van Vleck, Miss Mary Underhill, Mr. H. E. Webb, Professor R. S. Woodward.

The President of the Society, Professor Thomas S. Fiske, occupied the chair, being relieved by Professor E. W. Brown at the afternoon session. The Council announced the election of the following persons to membership in the Society: Miss G. C. Alden, Westfield, Mass.; Mr. L. D. Ames, University of Missouri; Professor R. C. Archibald, Ladies'. College, Sackville, N. B.; Mr. W. H. Bates, Purdue University; Miss H. D. Buckingham, Lexington, Mass.; Miss L. D. Cummings, Vassar College; Mr. Harry English, Washington, D. C.; Professor G. A. Gibson, Glasgow and West of Scotland Technical College; Miss M. F. Gould, Everett, Mass.; Dr. O. D. Kellogg, Princeton University; Mr. W. A. Manning, Stanford University; Dr. C. M. Mason, Massachusetts Institute of Technology; Professor Helen A. Merrill, Wellesley College; Mr. E. A. Miller, Massachusetts Institute of Technology; Mr. E. H. Taylor, Eastern Illinois State Normal School; Professor Anna L. Van Benschoten, Wells College; Mr. R. E. Wilson, Northwestern University. Nine applications for membership in the Society were received.

A list of nominations for officers and other members of the Council was adopted and ordered placed on the official ballot for the annual election. The office of assistant secretary of the Society was revived and filled by the appointment of Dr. William Findlay. A committee was appointed to make arrangements for holding the next summer meeting of the Society at St. Louis, and to coöperate with the committee of the St. Louis Exposition in organizing the Mathematical Section of the International Congresses. A committee consisting of Professors Joseph Bowden and Gustave Legras was appointed to audit the accounts of the treasurer.

The following papers were presented at this meeting:

(1) Dr. A. S. GALE: "On three types of surfaces of the third order regarded as double surfaces of translation."

(2) Dr. L. P. EISENHART: "Surfaces of Bonnet and their transformations."

(3) Dr. EDWARD KASNER: "On partial geodesic representation."

(4) Professor F. N. COLE: "On the factoring of large numbers."

(5) Professor E. GOURSAT: "A simple proof of a theorem in the calculus of variations" (extract from a letter to Professor W. F. Osgood).

(6) Mr. BURKE SMITH: "On the deformation of surfaces whose parametric lines form a conjugate system."

(7) Professor G. A. MILLER: "On the number of sets of conjugate subgroups."

(8) Mr. ELIJAH SWIFT: "On the condition that a point transformation of the plane be a projective transformation."

(9) Miss I. M. SCHOTTENFELS: "On the simple groups of order $8!/2$" (preliminary communication).

(10) Miss I. M. SCHOTTENFELS: "The necessary condition that two linear homogeneous differential equations shall have common integrals."

Professor Goursat's paper was communicated to the Society through Professor Osgood, Mr. Swift's paper through Professor Bôcher. In the absence of the authors, the papers of Professor Goursat and Mr. Swift were read by Professor Osgood, and Professor Miller's paper was read by title. Professor Cole's paper appeared in the December number of the BULLE-

TIN. Dr. Gale's paper appears in the present number. Abstracts the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

2. In seeking to determine the surfaces admitting a deformation with preservation of the lines of curvature Bonnet found that they are characterized by the property of having the same spherical representation of their lines of curvature as pseudospherical surfaces. Dr. Eisenhart considers these surfaces of Bonnet without any reference to their rôle in the theory of deformation and discovers certain transformations of these surfaces into surfaces of the same kind. When a configuration upon the sphere of the kind considered is given, the determination of all the surfaces of Bonnet with this representation of the lines of curvature requires the integration of an equation of Laplace; all the corresponding surfaces are said to form a group. The moulded surfaces and all the parallels of pseudospherical surfaces are surfaces of Bonnet. A determination is made of all surfaces of Bonnet with plane lines of curvature in one or two systems, and it is shown that the transformation of Lie for pseudospherical surfaces can be generalized so that, when the spherical representation of one group of surfaces of Bonnet is known, the representation of an infinity of other groups can be found directly.

Bianchi has shown that a pseudospherical surface can be transformed into an infinity of surfaces of the same kind whose tangent planes are perpendicular to the tangent plane of the given surface, and that the complete determination of this transformation requires the solution of a Riccati equation. It can be shown that this is only a special case of a similar transformation admitted by all surfaces of Bonnet, and the analytic work is the same as in the particular case. Each of these transforms belongs to a different group, but the determination of all the members of these groups requires the integration of one equation of Laplace. From each point M of a given surface S there diverge an infinity of planes tangent to the different transforms; the points of contact of these planes with the latter lie on the circumference of a circle in a plane parallel to the tangent plane to S and with the normal at M for axis; this gives a geometric interpretation of the cyclic systems associated with every surface of Bonnet.

Bianchi has considered the class of surfaces defined by the property: the spheres described on every segment of the normal comprised between the two centers of curvature as diameter cut a fixed sphere in great circles, or orthogonally, or all pass through the center. It is found that these surfaces are surfaces of Bonnet and their transforms also have the above property.

Bäcklund has shown that the transformation of Bianchi can be generalized so that for each angle σ there is an infinity of transforms of a pseudospherical surface such that the tangent planes to the given surface and the transform make the angle σ. This transformation also can be generalized so as to apply to all surfaces of Bonnet and the problem requires at most the solution of a Riccati equation. The points of contact of all transforms corresponding to a given σ lie in a circle whose axis is normal to S at M. If the problem has been completely solved for a given surface S, and if S_1 and S_2 are two of its transforms corresponding to angles σ_1 and σ_2, there is a transform of S_1 of angle σ_2 coinciding with a transform of S_2 of angle σ_1, and its determination requires only algebraic processes. From this we have that when all the transforms of a given surface can be found, the determination of the transforms of the transforms requires algebraic processes alone. These transformations are applied to particular surfaces of Bonnet and their transforms are considered.

3. Dini and Lie solved completely the problem of geodesic representation, which consists in the determination of all pairs of surfaces S, S_1 capable of point to point correspondence in such a manner that all the ∞^2 geodesics of S correspond to those of S_1. Dr. Kasner shows that the same surfaces are obtained if four systems of ∞^1 geodesics on each surface are to correspond. A more general problem arises, however, when merely three such systems are to correspond. (As to two systems, no problem arises, since then S and S_1 are entirely arbitrary.) This problem of *partial* geodesic representation is discussed for the conformal case and for the case where one of the surfaces is a plane, yielding extensions of recent results of Finsterwalder and Stäckel. The paper concludes with the explicit determination of all surfaces capable of conformal representation on the plane so that a system of (real) geodesics become straight lines.

5. The theorem given by Professor Osgood in the *Trans-*

actions, volume 2 (1901), page 273: "On a fundamental property of a minimum in the calculus of variations, etc." depends for its proof on the approximate determination of a limit for the value of the integral

$$\int_a^l [f'(x)]^2 dx, \qquad (a < l \leqq b);$$

more precisely, it is necessary to show that the value of the integral is greater than a certain positive quantity depending on a, b and $|f(l) - f(a)| = L$. Professor Goursat gives an exceedingly simple proof that

$$\int_a^l [f'(x)]^2 dx \geqq \frac{L^2}{l - a}.$$

6. Cosserat and Bianchi have given the necessary and sufficient condition that a surface should be susceptible of deformation so that a conjugate system of lines should remain conjugate during the deformation. Instead of a single surface, we may think of a continuous system of surfaces, which represent the different forms into which the single surface is deformed.

In Mr. Smith's paper, after proving Bianchi's theorem that surfaces which possess the property mentioned are in every case the associates of surfaces whose curvature, expressed in terms of the parameters of their asymptotic lines, is of the form

$$K = -\frac{1}{[\phi(u) + \chi(v)]^2},$$

two surfaces whose curvature is of this type are considered. They are the right conoid and a ruled surface of the third order, and it is shown that the deformation problem can be completely solved for these surfaces by considering their spherical representations. This leads to two classes of surfaces which may be deformed so that a conjugate system of lines remains a conjugate system; one of these, the associate of the conoid, was found by Goursat in 1891 by other methods, and the other has for its equations

$$x = \frac{\partial}{\partial u}\left[(U + V - vV') \cdot \frac{1 - 3u^2}{1 + u^2}\right] - 3V',$$

$$y=\frac{\partial}{\partial u}\left[(U+V-vV')\cdot\frac{1-3u^2}{1+u^2}\right]+3V',$$

$$z=\sqrt{2}\frac{\partial}{\partial u}\left[(U+V-vV')\cdot\frac{u(3-u^2)}{1+u^2}\right].$$

The properties of the latter surfaces are discussed and it is shown how the other members of the continuous system may be found by Bonnet's theory.

7. Professor Miller's paper is in the abstract as follows: According to Sylow's theorem there is only one set of conjugate subgroups of order p^α in a group G of order $p^\alpha n$, where p is any prime number and n is not divisible by p. The group G contains at least one subgroup of order p^β, $\beta<\alpha$. If m represents the number of sets of subgroups of order p^β which are contained in a particular subgroup of order p^α and are formed by combining into one set all of these subgroups of order p^β which are conjugate under G, then the number of the sets of conjugate subgroups of order p^β in G cannot exceed m. The number m can clearly not exceed the number of sets of conjugate subgroups of order p^β in one of the subgroups of order p^α. The latter number depends upon the type of the subgroup of order p^α, otherwise it is independent of the type of G.

In particular, if the subgroups of order p^α are cyclic, all the subgroups of order p^β which are found in G form a single conjugate set. Moreover, if a group of order p^α contains only one subgroup of order p^β, all the subgroups of order p^β in G form a single conjugate set. As β may equal α, this statement includes the fact that all the subgroups of order p^α form a single conjugate set under G. If $p^\alpha=4$, G cannot contain more than three sets of conjugate subgroups of order 2. It is proved that there cannot be just two such sets of conjugate subgroups in G. If G contains contains three sets of conjugate operators of order 2 it must contain a characteristic subgroup of order m which is composed of all its operators of odd order.

8. Möbius is commonly stated to have proved the theorem that any point transformation of the plane which is a collineation is a projective transformation. An examination of Möbius's work, however, shows that, besides the assumption that we are dealing with a one to one point transformation which is

a collineation, certain other assumptions are tacitly made. The most obvious of these is the assumption that the transformation is continuous. Moreover, if we are to get anything beyond the affine transformations, we cannot assume that the transformation is defined at every point of both planes, since there are some points which will be "thrown to infinity" and therefore, strictly speaking, have no images in the other plane. It is the object of Mr. Swift's paper to establish this theorem of Möbius without the assumption of continuity and with as slight assumptions as possible concerning the regions for which the transformation is defined. The theorem proved takes the following form: Two plane point sets being given, subject to the sole restriction that each contains interior points, if it is possible to establish a one to one correspondence between the points of these sets of such a character that to any three collinear points of either set correspond three collinear points of the other, then this correspondence can be effected by a projective transformation. It should be noticed that here, as also in Möbius's work, no reference is made to imaginary points.

9. Miss Schottenfels's preliminary communication contains the proof that all types of simple groups of order $8!/2 = 20160$ separate, with respect to the number of conjugate subgroups of order 5, into two classes containing 336 or 2016 conjugate subgroups of order 5, respectively.

10. Professor Von Escherich in the *Denkschriften der Wiener Akademie*, volume 46, and Heffter in *Crelle's Journal*, volume 116, proved that there exists for linear differential homogeneous equations a theory analogous to that of algebraic equations, and this subject has recently been revived by Drs. Epsteen and Pierce in the *American Mathematical Monthly* of 1903.

The paper offered by Miss Schottenfels is supplementary to those of the last mentioned authors, and establishes the necessary condition that two linear homogeneous differential equations shall have two or more common integrals.

The theorem is illustrated by the following application to two linear homogeneous differential equations:

$$(1) \quad \alpha_0(x)\frac{d^4y}{dx^4} + \alpha_1(x)\frac{d^3y}{dx^3} + \alpha_2(x)\frac{d^2y}{dx_4} + \alpha_3(x)\frac{dy}{dx} + \alpha_4(x)\cdot y = 0,$$

$$(2) \qquad \beta_0(x)\frac{d^3y}{dx^3} + \beta_1(x)\frac{d^2y}{dx_4} + \beta_2(x)\frac{dy}{dx} + \beta_3(x)\cdot y = 0.$$

The necessary condition that (1) and (2) have two integrals in common is that, in the following matrix obtained by successive differentiation,

$$\left\{\begin{matrix} \alpha_0 & \alpha_0' + \alpha_1 & \alpha_1' + \alpha_2 & \alpha_2' + \alpha_3 & \alpha_0' + \alpha_4 & \alpha_4' \\ 0 & \alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_0 & 2\beta_0'+\beta_1 & \beta_0''+2\beta_1'+\beta_2 & \beta_1''+2\beta_2'+\beta_3 & \beta_2''+2\beta_3' & \beta_3'' \\ 0 & \beta_0 & \beta_0'+\beta_1 & \beta_1'+\beta_2 & \beta_2'+\beta_3 & \beta_3' \\ 0 & 0 & \beta_0 & \beta_1 & \beta_2 & \beta_3 \end{matrix}\right\},$$

the determinant consisting of the first five columns, and also that consisting of the first four columns and the sixth, shall vanish identically.

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TWO SYSTEMS OF SUBGROUPS OF THE QUATERNARY ABELIAN GROUP IN A GENERAL GALOIS FIELD.

BY PROFESSOR L. E. DICKSON.

(Read before the American Mathematical Society, August 31, 1903.)

1. CONSIDER first the group G_ω composed of the

$$\omega = p^{4n}(p^{2n}-1)(p^n-1)$$

operators of the homogeneous quaternary abelian group in the $GF[p^n]$, $p > 2$, which multiply the variable η_1 by a constant. Those of its operators which leave ξ_1 and η_1 unaltered are given the notation

$$\begin{bmatrix} a & \gamma \\ \beta & \delta \end{bmatrix} : \quad \begin{matrix} \xi_2' = a\xi_2 + \gamma\eta_2, \\ \eta_2' = \beta\xi_2 + \delta\eta_2, \end{matrix} \qquad (a\delta - \beta\gamma = 1).$$

Certain other operators of G_ω are given the notation

$$[k, a, c, \gamma] = \left\{\begin{matrix} 1 & k & a & c \\ 0 & 1 & 0 & 0 \\ 0 & c-\gamma a & 1 & \gamma \\ 0 & -a & 0 & 1 \end{matrix}\right\}$$