tion. The principal addresses have already been given in outline in the BULLETIN,* with the exception of Mittag-Leffler's account of the correspondence between Weierstrass and S. Kowalevski. This is perhaps as interesting as anything in the book. At the end of the volume there is given, in a French translation, a paper by Veronese, "Les postulats de la géométrie dans l'enseignement," which was intended for the Congress, though it was not actually presented then. Among the shorter articles the most satisfying and appropriate are those of the character of an aperçu, as for instance d'Ocagne, "Sur les divers modes d'application de la méthode graphique à l'art du calcul"— Padé, "Aperçu sur les développements récents de la théorie des fractions continues"— and Amodeo, "Coup d'œil sur les courbes algébriques au point de vue de la gonalité." Amodeo, in particular, has given a most interesting résumé of results obtained regarding groups of points, g_k^1, cut out on a curve of order m by adjoints of order $m-3-\alpha$, adjoints whose existence presupposes the condition $p \geqq \frac{1}{2}\alpha m + 1$. But different readers will naturally find most to interest them in different papers.

CHARLOTTE ANGAS SCOTT.

Probabilités et Moyennes Géométriques. Par EMANUEL CZUBER. Traduit par HERMANN SCHUERMANS. Préface de CHARLES LAGRANGE. Paris, A. Hermann, 1902. xi + 214 pp.

THE various problems considered in the direct theory of probability may be roughly divided, as regards the number of possible cases, into four classes. First, the ordinary problems arising in connection with games of chance, where the number of cases is limited, the methods employed being those of combinatorial analysis. Second, problems where the number of cases is still finite, but very large, so that recourse must be made to approximate results based upon Stirling's formula. Here belong, for example, Bernouilli's theorem and the so-called law of large numbers. The third class includes those problems in which the number of cases is infinite, depending upon the values of a certain number of arbitrary parameters; while in a fourth class we may place the problems depending upon arbitrary laws or functions. The last class has as yet received little attention. It may be remarked in passing that

* BULLETIN, vol. 7 (1900-1901), pp. 57-79.

there is another type, intermediate between the second and third, in which the number of cases is infinite, but forms merely a denumerable assemblage. Such problems arise naturally in connection with the ordinary theory of numbers, but so far as the reviewer knows have never been considered.

Professor Czuber's well known monograph, now appearing in an attractive French translation, is devoted to the third class of problems, and the related questions of mean value. The term geometric or local probability is justified, as is remarked in the introduction, in view of the fact that if a problem depends upon arbitrary parameters, it is possible to translate it into an equivalent geometric problem relating to points or more complicated configurations taken at random in certain regions. The three chapters on local probability treat respectively 1° Points taken at random on lines or surfaces or in space ; 2° Lines taken at random in a plane or in space; 3° Planes taken at random. The discussion of mean value, forming the second part of the book, is much briefer, occupying only a single chapter of sixty pages.

The questions considered in this youngest branch of the theory of probability have for their type the famous needle problem of Buffon, proposed and solved by the latter in his Essai d'arithmétique morale, published 1777 in the fourth volume of the Supplément a l'histoire naturelle, but composed some years earlier. Laplace in his Théorie, pages 359–362, treated the same problem, without however referring to Buffon. It may be remarked that the latter proposed also a more complicated problem, where the plane is ruled into congruent rectangles by means of two sets of parallels, but his solution is incorrect. Laplace gave the solution for the case where the needle or rod is shorter than both dimensions of the rectangle. Todhunter in his History of the mathematical theory of probability, § 650, considers the more general problem, but his results, according to Czuber, are incomplete. Since Laplace the chief contributions to the subject are due to Barbier, Jordan and Lalanne of the French school, and McColl, Sylvester and Crofton of the English school.

Of especial interest are the accounts, in connection with some of the problems discussed by Professor Czuber, of experiments verifying the theoretical results. Thus in connection with Buffon's problem, Professor Wolf in 1850, using a rod four fifths the distance between the parallels, found that in 5000 trials there were 2532 cases of intersection. The theoretical probability in this case is $8/5\ \pi$, so that the probable value of π, obtained by comparison with the experimental probability, is 3.1596. No mention however is made of a similar series of experiments reported by De Morgan in his Budget of Paradoxes, page 172, leading to the still more accurate value 3.1412. It would be of interest if some one would undertake to carry out the experimental rectification of curves suggested on page 116 of Czuber's book, and based upon Crofton's theorem that the number of random lines which intersect a closed convex curve is proportional to the length of the curve.

It is a commonplace to remark that in no branch of mathematics is it so easy to make errors as in the theory of probability. The development of the theory is marked by a continual revision of the results of earlier investigators. Especially is this true of geometrical probability, where the essential difficulty lies in the definition of what is to be regarded as equally possible, or, as Venn happily expresses it, in "the idealization of the randomness." Mathematically this amounts to fixing the independent parameters which are to be regarded as equicrescent in the problem considered. In the present book only one error of any importance has come to the notice of the reviewer. This occurs in finding the probability that three segments, each less than a given limit, shall determine an acute or obtuse triangle respectively (pages 29, 30). The results obtained, $1 - \pi/4 = .216$ and $\pi/4 = .784$, cannot both be correct since there is a third case, not considered in the text, where the three segments do not determine any triangle. The error is not corrected in the translation, so it may be worth while to give here the correct results, which are $1 - \pi/4 = .216$, $\pi/4 - \frac{1}{2} = .285$, $\frac{1}{2} = .500$, respectively, for the three cases mentioned.

The English student finds introductions to the subject of local probability in the excellent chapters in Todhunter's and Williamson's Integral Calculuses (the latter chapter written by Professor Crofton); but of course these cannot compare in completeness with a monograph. The present translation should therefore prove welcome to the large class of students who find it more economical to read French than German.

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