itself is commutative with every operator of G_i. Let H_1 be the commutator subgroup. The group $\{H_1, G_i\}$ is of order $p_1^\beta p_i^{\alpha_i}$. This contains $p_1^\gamma (\gamma \leqq \beta)$ subgroups of order $p_i^{\alpha_i}$, and therefore $p_1^\gamma \equiv 1 \pmod{p_i}$. Hence if

$$p_1^\gamma \not\equiv 1 \pmod{p_i} \quad (0 < \gamma \leqq \beta),$$

every commutator is commutative with every operator of G_i. Then $A_j^{-1} A_i A_j = A_i t_i$, where A_j is any operator of

$$G_j \qquad (j = 1, 2, \cdots, n)$$

and A_i is any operator of G_i; and $A_j^{-1} A_i^{p_1^{\beta_i}} A_j = A_i^{p_1^{\beta_i}}$, where $p_1^{\beta_i}$ is the order of t_i. But $p_1^{\beta_i}$ is relatively prime to p_i. Therefore $A_j^{-1} A_i A_j = A_i$, and G is the direct product of the groups G_j."

THEOREM. *If a group G of order $p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_n^{\alpha_n}$ ($p_1, p_2, \cdots, p$. being distinct primes) has a commutator subgroup of order p_1^β and if $p_1^\gamma \not\equiv 1 \pmod{p_i}$ $(0 < \gamma \leqq \beta)$, $(i = 2, 3, \cdots n)$, then G is the direct product of groups of orders $p_1^{\alpha_1}, p_2^{\alpha_2}, \cdots, p_n^{\alpha_n}$ respectively.*

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NOTE ON IRREGULAR DETERMINANTS.

BY PROFESSOR L. I. HEWES.

IN Gauss's table * of binary quadratic forms the two negative determinants -468 and -931 of the first thousand are classed as regular and their genera and classes given correctly. Perott † has pointed out that these two determinants are irregular. The details of the classes of the original thirteen irregular determinants of Gauss have been worked out by Cayley ‡ and on the following page are given the details, in his notation, for the properly primitive reduced forms of the two determinants added by Perott's investigation.

* C. F. Gauss, Werke, vol II, p 450.
† "Sur la formation des déterminants irreguliers," *Crelle*, vol. 59.
‡ Cayley's Collected Papers, vol. 5, p. 141.

D	Classes.			α	β	δ	Cp.
— 468	1	0	468	+	+	+	1
	13	0	36				e^2
	9	0	52				e_1^2
	4	0	117				$e^2e_1^2$
	9	3	53	—	+	+	e_1
	17	—5	29				e^2e_1
	17	5	29				$e^2e_1^3$
	9	—3	53				e_1^3
	19	8	28	+	—	—	e
	7	—1	67				$e^3e_1^2$
	7	1	67				ee_1^2
	19	—8	28				e^3
	8	2	59	—	—	—	e^3e_1
	11	—4	44				ee_1
	11	4	44				$e^3e_1^3$
	8	—2	59				ee_1^3
							Exponent of irregularity 2.

D	Classes.			α	β	Cp.
— 931	1	0	931	+	+	1
	4	1	233			g^4
	4	—1	233			g^2
	25	12	43			dg^4
	25	— 12	43			d^2g^2
	11	2	85			d
	11	—2	85			d^2
	23	9	44			dg^2
	23	—9	44			d^2g^4
	19	0	49	+	—	g^3
	17	2	55			g
	17	—2	55			g^5
	5	—2	187			dg^5
	5	2	187			d^2g^5
	20	3	47			dg^3
	20	—3	47			d^2g^3
	20	7	49			d^2g
	20	—7	49			dg^5
						Exponent of irregularity 3.

KINGSTON, R. I.,
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