Type VIa.

$$(X_1, X_2) \equiv (X_2, X_3) \equiv (X_3, X_1) \equiv 0,$$

$$(X_1, X_4) \equiv 0, \quad (X_2, X_4) \equiv 0, \quad (X_3, X_4) \equiv 0.$$

$$A_1 \equiv \frac{\partial}{\partial a_1}, \qquad A_2 \equiv \frac{\partial}{\partial a_2}, \qquad A_3 \equiv \frac{\partial}{\partial a_3}, \qquad A_4 \equiv \frac{\partial}{\partial a_4}.$$

UNIVERSITY OF CINCINNATI,
October, 1901.

SHORTER NOTICES.

Einführung in die Theorie der Differentialgleichungen mit einer unabhängigen Variablen. Von Dr. LUDWIG SCHLESINGER, ordentlichem Professor an der Universität zu Klausenburg. Leipzig, Göschen, 1900. Pp. viii + 310.

THIS little volume, which forms part of the "Sammlung Schubert" (cf. the BULLETIN for January, 1901, p. 192), gives, we believe, the best introduction which has yet appeared to that important side of the theory of ordinary differential equations in which the points of view are those of the theory of functions of a complex variable. Thus the discussion of the nature of singular points holds a central position in the treatment given. The author has been particularly successful in his choice of topics. He has on the one hand restricted himself to the simpler parts of the subject, more than half the volume being devoted to linear differential equations of the second order, and the remainder to the case of a single equation of the first order. By doing this he has succeeded in avoiding long analytical developments which only confuse a beginner without really teaching him anything. On the other hand the author has treated these simple cases in such a way as to bring out clearly a large number of the most important points of view of the modern theory of differential equations. Some of Dr. Schlesinger's own investigations, to mention only one point, on the Laplacian and Eulerian transformations are here set forth in particularly attractive form, although, of course, only for very special differential equations. The reader can turn to a large treatise for further information on these or other questions if he has once become interested in them, and a great merit of this book is that he will almost inevitably become interested in many of the subjects treated.

We have noticed the absence of only one important subject which would seem to belong in a treatment of this sort. We refer to the theory of the conformal transformation effected by the ratio of two solutions of a homogeneous linear differential equation of the second order (Schwarz's s-function). The volume before us does not, of course, touch those sides of the subject with which Lie's name is connected, nor is any special attention devoted to the theory of the real solutions of differential equations, but in those parts of the theory which he professes to treat the author has achieved more than ordinary success.

MAXIME BÔCHER.

Ιωάννου Ν. Χατζιδάκι — Εἰσαγωγὴ εἰς τὴν Ἀνωτέραν Ἀλγέβραν. Second edition. Athens, 1898.

THE work of Professor Hatzidakis is a scientific development by Weierstrass's methods of the general principles of arithmetic which form the basis of algebra, of calculus, and of the theory of functions. The natural numbers are defined by the process of counting and in the first chapter is placed the *formal* reasoning by which the associative and commutative laws are deduced from the special cases

$$\beta + \alpha = \alpha + \beta, \tag{1}$$

$$\alpha + (\beta + \gamma) = \alpha + \beta + \gamma. \tag{2}$$

The second chapter deals with fractional numbers, which are regarded as collections of fractional units, and the fraction $\frac{\alpha\beta}{\beta}$ is equal to the natural number α. Then any two numbers of the enlarged system are said to be equal if integral equimultiples of them are equal. Multiplication of fractional numbers is defined according to the distributive law, *i. e.*, 1.41×1.73 is the sum of the products obtained by multiplying every one of the units of the one factor by every unit of the other. The product of two units is defined in like manner so that equals multiplied by equals shall give equals. Thus, since we wish the two products $\frac{10}{10} \times \frac{100}{100}$ and 1×1 to be equal and since

$$\tfrac{10}{10} \times \tfrac{100}{100} = 1000\,(\tfrac{1}{10} \times \tfrac{1}{100}),$$

the product $\frac{1}{10} \times \frac{1}{100}$ is defined to be $\frac{1}{1000}$. From this