find the book well worth reading if for no other reason than to look at the theory from another and entirely different aspect.

G. O. JAMES.

Leçons sur la Théorie des Formes et la Géométrie analytique supérieure. Par H. ANDOYER. Volume I. Paris, Gauthier-Villars, 1900. 8vo, pp. vi + 508. Price, 15 francs.

IT would be asserting too much to say that M. Andoyer's first volume fulfills all reasonable expectations. In this volume only binary and ternary forms are treated, the quaternary field being reserved for a second, which is announced as already in press. All who read the author's introduction (lithographed) a few years ago must have expected to find in the present work a treatise not only compendious but also elementary. Compendious it certainly is, covering a surprisingly wide range, but the student beginning in this subject and reading it alone will find it impenetrable. It is lucid, it is concise, but it is extremely condensed. Therein lies, however, its great merit. As a work of reference, or as a syllabus to accompany a lecture course, it will supersede anything hitherto published in the same field. Its field is geometry—invariantive geometry, algebra taking the second place. Accordingly one cannot yet dispense with Salmon, Faa di Bruno, and Elliott. Nor is it intended to precede the study of projective geometry of curves; rather it presupposes a large amount of geometrical knowledge, and aims to recast and systematize it. To quote from the preface: "Je me suis proposé, en l'écrivant, d'exposer d'une façon didactique la théorie des formes et son interprétation géométrique générale."

Binary forms are treated in ten chapters, occupying 145 pages. After two excellent but too compact preliminary chapters, linear and quadratic forms and systems of forms are fully discussed, together with formal treatment of resultants and discriminants. Cubics, quartics, and quintics with their full covariant systems are given, the last only in list without any detail. All systematic discussion of *complete* form systems is excluded, the reader being referred to Gordan and Hilbert. Finally forms in two sets of variables are taken up, otherwise correspondences, and the metric geometry on a line. The chapter most novel in this binary division is that on the doubly quadratic form, or the (2, 2) correspondence on a line. Of course the problem of derived correspondences (2, 2) and the conditions for the occurrence of closed systems are of principal importance here, and I believe that no algebraic treatment has been accessible in textbooks heretofore. Andoyer derives the explicit fundamental invariants and covariants, the recursive formula expressing the equation of the $(n+1)$th derivative correspondence in terms of the nth and $(n-1)$th, and so ultimately in rational covariants of the original form. The results here obtained are directly applied in the chapter on Poncelet's polygons, in the ternary division.

Under ternary forms occur the necessary formal generalities, short chapters on homography and on conics (*la sêrie quadratique*); then that just now alluded to upon the system of two conics, a welcome addition to the literature of the subject. Invariant conditions for closed Poncelet polygons of 3 and of 4 sides are given in rational functions of the four fundamental invariants, and the apparatus is furnished for calculating similar conditions for n sides. The derivation of these results by the aid of the elementary invariants of a binary biquadratic form is a decided improvement upon older methods.

Two bilinear forms and the quadratic correspondence are handled thoroughly, with a view to the quartic curve. Cubic and trilinear forms have a fairly full treatment (53 pages). The quartic curve is considered as the envelope of a variable conic whose parametric point describes a fixed conic: thus the bitangents and quadruply tangent conics are examined by the method of Cayley and Salmon. The two concluding chapters, apparently valuable, are concerned with metric geometry, general and special, *i. e.*, with a proper quadric and a degenerate quadric respectively as the absolute.

The thoroughly systematic execution of the work perhaps excuses the absence of any index, and the geometrical purpose naturally excludes any account of the work of Hilbert or of Deruyts; but a work of this magnitude, not professedly a cyclopædia, ought certainly to offer some perspectives and some outlook. The promise of a general bibliography in Volume II is hardly a substitute for the minute and particular citations that a student needs.—Enough of fault-finding! It is refreshing to read a treatise that makes perfectly free use of invariants of every degree of intricacy, with an eye solely to their significance and no concern about their explicit formulation. Let us notice that this last product of the nineteenth century on plane curve theory stops short of the quintic.

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