

are not enumerated in any  $x$ -group; thus two points of a  $x$ -group fall at  $P$ , in whatever direction  $P$  is taken. This may perhaps be most simply expressed by saying that the complement to  $A$  contains an evanescent circuit about  $A$ . This is what de Paolis refers to as a "punto fondamentale doppio"; it is one variety of the fundamental point of the third species.

The fundamental points and lines, the double points, and the Jacobian, in the first plane, with their correspondents in the second plane, form a kind of framework for the transformation, indicating everything exceptional; and a complete knowledge of them is essential to a full comprehension of the possible effects of the general rational transformation. If  $x > 2$ , the co-jacobian has to be taken into account also, and as I have here shown, the different species of fundamental points and lines can no longer be discriminated; hence the purely geometrical treatment becomes very involved, and it appears necessary to resort to analysis. Thus it is hardly to be expected that the treatment of transformation can be carried any further on the lines of Cremona and de Paolis; but the intrinsic interest of de Paolis' work is surely excuse enough for devoting some little space to it in a periodical of a critical and historical nature.

BRYN MAWR COLLEGE,  
May, 1900.

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#### NOTES.

THE Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will meet in the Ryerson physical laboratory of the University of Chicago, on Thursday and Friday, December 27th and 28th next. Titles and abstracts of papers to be read at this meeting should be in the hands of the Secretary of the Section not later than December 5th.

THE third (July) number of the *Transactions* of the AMERICAN MATHEMATICAL SOCIETY contains the following papers: "Wave propagation over non-uniform conductors," by M. I. PUPIN; "Ueber Systeme von Differentialgleichungen denen vierfach periodische Functionen Genüge leisten," by M. KRAUSE; "On linear criteria for the determination of the radius of convergence of a power series," by E. B. VAN VLECK; "On the existence of the Green's function for the

most general simply connected plane region," by W. F. OSGOOD; "'D' lines on quadrics," by A. PELL; "Sundry metric theorems concerning  $n$  lines in a plane," by F. H. LOUD; "An application of group theory to hydrodynamics," by E. J. WILCZYNSKI; "Determination of an abstract simple group of order  $2^7 \cdot 3^6 \cdot 5 \cdot 7$  holoedrally isomorphic with a certain orthogonal group and with a certain hyperabelian group," by L. E. DICKSON.

In the October (closing) number of volume I. of the *Transactions* it is proposed, for the convenience of the readers, to collect under an appropriate heading brief notes concerning the memoirs of the volume. For this purpose the contributors are requested to furnish a complete list of errata in their memoirs as published, and any further notes, for instance in the way of reference.

THE German mathematical society held its annual meet-at Aachen, September 16th–23d, 1900, with Professor D. HILBERT as president, and Professor A. GUTZMER as secretary. The preliminary programme of papers to be presented is as follows:

(1) W. EBERT, Paris: "On the direct measurement of the motion of the earth's axis of rotation by means of stars near the pole."

(2) E. JÜRGENS, Aachen: "Computation of determinants."

(3) F. KLEIN, Göttingen: "Mechanics in the Encyclopædia of the mathematical sciences."

(4) A. KNESER, Dorpat: "On the development and present state of the calculus of variations."

(5) E. KÖTTER, Aachen: Subject to be announced.

(6) A. MARCUSE, Berlin: "New development of the theory and practice of the determination of geographical and nautical positions."

(7) A. MARCUSE, Berlin: "A new photographic universal instrument for the determination of geographic-astronomical positions."

(8) F. MEYER, Königsberg: "On singular binary forms and relations between sub-determinants."

(9) F. MEYER, Königsberg: "On geometrical theorems of the nature of those of Pascal and Desargues."

(10) H. MINKOWSKI, Zürich: "On the concepts: length, surface, and volume."

(11) P. H. SCHOUTE, Groningen: "On the number of points, right lines, planes, etc., of space of  $n$  dimensions."

(12) H. SCHUBERT, Hamburg: "Zur Schlick'schen Ausgleichung der Schiffsvibrationen."

(13) D. SINTZOW, Ekaterinoslaw: "On connexes in space."

(14) P. STÄCKEL, Kiel: "On the theory of geodetic lines."

(15) A. WANGERIN, Halle: "Proof of a theorem relative to lines of curvature."

(16) A. WANGERIN, Halle: "Determination of all surfaces having constant curvature."

GAUTHIER-VILLARS announces the recent publication of *Leçons sur la théorie des formes et la géométrie analytique supérieure*, volume I, by H. ANDOYER; *Recueil de problèmes de géométrie analytique*, by FRANCOIS MICHEL; *Analyse infinitésimale à l'usage des ingénieurs*, volume I, by E. ROUCHÉ and L. LÉVY. The same publishing house has in preparation a translation into French of the *Encyclopædia der mathematischen Wissenschaften*, the first three volumes of which are in the editorial charge of JULES MOLK. The first volume to appear in print is volume II of the work which is expected from the press early in 1901. Among other works in preparation are a *Cours d'analyse mathématique*, by E. GOURSAT and an *Essai sur les fondements de géométrie* by W. B. RUSSELL. The second volumes of the above-named works of ANDOYER and of ROUCHÉ and LÉVY are in press.

THE Macmillan Company have in preparation an Introduction to celestial mechanics by Dr. F. R. MOULTON, associate professor of astronomy in the University of Chicago, and *The principles of mechanics*, by Professor FREDERICK SLATE, of the University of California.

THE several universities below offer during the winter semester 1900-1901 courses in mathematics as follows:

UNIVERSITY OF BERLIN.—By Professor L. FUCHS: Theory of elliptic functions, four hours; Introduction to the theory of differential equations, four hours; Seminar, two hours.—By Professor H. A. SCHWARZ: Differential calculus, four hours; Applications of the theory of elliptic functions, two hours; Calculus of variations, four hours; Colloquium, two hours; Seminar, two hours.—By Professor G. FROBENIUS: Theory of algebraic equations, four hours; Seminar, two hours.—By Professor J. KNOBLAUCH: Analytical geometry, four hours; Definite integrals, four hours; Exercises, one hour.—By Professor K. HENSEL; Integral cal-

culus, four hours; Theory of numbers, four hours; Colloquium, two hours.—By Professor R. LEHMANN-FILHÉS: Analytical mechanics, four hours; Exercises, one hour.

UNIVERSITY OF BONN.—By Professor L. HEFFTER: Analytical geometry, four hours; Elliptic functions, four hours; Exercises in geometry, one hour.—By Professor H. KORTUM: Infinitesimal calculus II, four hours; Curved lines and surfaces, two hours; Mathematical seminar, two hours.—By Professor R. LIPSCHITZ: Analytical mechanics, four hours; Mathematical seminar, two hours.

UNIVERSITY OF BresLAU.—By Professor J. ROSANES: Algebraic equations, four hours; Elements of the theory of differential equations, two hours; Seminar, one hour.—By Professor R. STURM: Infinitesimal calculus, four hours; Theory of numbers, two hours; Seminar, one hour.—By Dr. F. LONDON: Analytical mechanics, four hours, with exercises, one hour.

UNIVERSITY OF ErlANGEN.—By Professor M. NOETHER: Infinitesimal calculus, four hours; Introduction to algebraic analysis and the theory of functions, four hours; Mathematical exercises.—By Professor P. GORDAN: Analytical Geometry, four hours; Algebra, four hours; Seminar.

UNIVERSITY OF Freiburg.—By Professor J. LÜROTH: Analytical mechanics, five hours; Trigonometry, three hours; Seminar, one hour.—By Professor L. STICKELBERGER: Plane analytical geometry, four hours; Theory of numbers, three hours.—By Dr. A. LOEWY: Differential calculus, three hours; Theory of surfaces, three hours; By Dr. E. REBMANN: Methodology of mathematical instruction, two hours.

UNIVERSITY OF GIESSEN.—By Professor M. PASCH: Infinitesimal calculus, four hours; Selected chapters of analytical geometry, two hours; Exercises in the elements of higher mathematics, one hour; Seminar, one hour.—By Professor E. NETTO: Analytical geometry of space, four hours; Algebraic equations, two hours; Seminar, one hour.—By Professor R. HAUSSNER: Introduction to the theory of differential equations, four hours; Technical mechanics, four hours; Theory of determinants, two hours.

UNIVERSITY OF GÖTTINGEN.—By Professor F. KLEIN: Projective geometry, four hours, with exercises, two hours.

—By Professor D. HILBERT: Theory of functions, four hours; Partial differential equations, four hours, with seminar in the latter, two hours.—By Professor M. BRENDEL: Rotational motion of the heavenly bodies, two hours; Exercises in numerical reckoning, two hours.—By Professor E. WIECHERT: Analytical mechanics, four hours.—By Professor F. SCHILLING: Integral calculus, four hours, with exercises, one hour.—By Professor AMBRONN: Method of least squares, two hours.—By Professor K. BOHLMANN: Mathematical principles of insurance, three hours; Mathematical statistics, two hours; Seminar in insurance, two hours.—By Dr. E. ZERMELO: Mengenlehre, two hours; Kinetic theory of gases, two hours.—By Dr. J. SOMMER: Theory of algebraic functions, two hours.

UNIVERSITY OF GREIFSWALD.—By Professor W. THOMÉ: Mechanics I, four hours; Algebra, four hours; Seminar, two hours.—By Professor E. STUDY: Infinitesimal calculus II, four hours; Theory of functions II, four hours; Seminar, two hours.

UNIVERSITY OF HALLE.—By Professor G. CANTOR: Infinitesimal calculus, five hours.—By Professor A. WANGERIN: Chapters in higher geodesy, one hour; Integral calculus with exercises, four hours; Applications of elliptic functions, two hours; Calculus of variations, two hours.—By Professor V. EBERHARD: Theory of algebraic equations, four hours; Numerical equations with exercises, one hour.—By Dr. H. GRASSMANN: Ordinary differential equations, two hours; Exercises in descriptive geometry, one hour.—By Dr. E. NEUMANN: Theory of continued fractions, two hours.

UNIVERSITY OF HEIDELBERG.—By Professor L. KOENIGSBERGER: Higher algebra, four hours; Elements of the theory of differential equations, two hours; Theory of numbers, two hours; Seminar, two hours.—By Professor H. VALENTINER: Method of least squares, two hours.—By Professor M. CANTOR: Infinitesimal calculus, four hours, with exercises, one hour; Political arithmetic, two hours.—By Professor F. EISENLOHR: Theoretical optics, four hours; Infinitesimal calculus, five hours; Potential, two hours.—By Professor K. KOEHLER: Synthetic geometry of space, three hours.—By Professor G. LANDSBERG: Descriptive geometry, four hours; Selected chapters of theory of functions, continued from preceding semester, two hours.—By Dr. K. BOEHM: The principles of Heinrich Hertz's

mechanics, two hours ; Gauss's memoir on the theory of surfaces, one hour.

UNIVERSITY OF INNSBRUCK.—By Professor O. STOLZ: Real differential and integral calculus, four hours ; Theory of functions of a complex variable after Cauchy and Weierstrass, three hours.—By Professor W. WIRTINGER: Linear partial differential equations, three hours ; Eulerian integrals, two hours ; Mathematical seminar, two hours.—By Dr. K. ZINDLER: Descriptive geometry with exercises, four hours ; Application of the differential calculus to geometry, two hours.

UNIVERSITY OF JENA.—By Professor J. THOMAE: Application of infinitesimal calculus to geometry, four hours ; Definite integrals and hypergeometric series, two hours ; Seminar, two hours.—By Professor A. GUTZMER: Integral calculus, four hours, with seminar exercises, one hour ; Elements of descriptive geometry with exercises, four hours.—By Professor G. FREGE: Analytical geometry of space, four hours ; Concepts, one hour.

UNIVERSITY OF KIEL.—By Professor L. POCHHAMMER: Application of infinitesimal calculus to geometry, three hours ; Introduction to the theory of functions, three hours ; Seminar, two hours.—By Professor P. STÄCKEL: Higher analysis II, four hours ; Stereometric drawing, four hours ; Calculus of variations, one hour ; Seminar, one hour.

UNIVERSITY OF KOENIGSBERG.—By Professor F. MEYER: Selected chapters of higher geometry, two hours ; Integral calculus, three hours, with exercises, one hour ; Seminar, one hour and a half.—By Professor A. SCHOENFLIES: Mechanics, four hours.—By Professor L. SAALSCHÜTZ: Theory of calculus of finite differences, two hours ; Introduction to algebraic analysis, three hours, with exercises, one hour.—By Dr. E. MÜLLER: Analytical geometry of space, two hours ; Perspective, two hours.

UNIVERSITY OF LEIPSIK.—Professor SCHEIBNER offers no courses this semester, and those of Professor MAYER will be announced later.—By Professor C. NEUMANN: Application of the infinitesimal calculus to geometry, four hours ; Seminar, one hour.—By Professor O. HÖLDER: Infinitesimal calculus, five hours ; Application of elliptic functions, one hour ; Mathematical seminar, one hour.—By Professor L. BOLTZMANN: Analytical mechanics, five hours.—By Professor F. ENGEL: Projective geometry and homogeneous co-

ordinates, three hours ; Differential invariants, two hours ; Seminar, one hour.—By Dr. F. HAUSDORFF : Theory of probability, three hours ; Map projection, two hours.—By Dr. G. KOWALWOSKI : Theory and application of determinants, two hours ; Theory of partial differential equations, two hours ; Seminar in differential invariants, in conjunction with Professor Engel, one hour.—By Dr. H. LIEBMAN : Elementary theory of numbers, two hours ; Foundations of geometry, two hours.

UNIVERSITY OF MARBURG.—By Professor F. SCHOTKY : Elliptic functions, four hours ; Algebraic analysis, three hours ; Seminar, two hours.—By Professor E. HESS : Integral calculus, five hours ; Selected chapters of geometry, three hours ; Spherical trigonometry, two hours ; Seminar, two hours.—By Dr. F. v. DALGWIK : Analytical geometry of the plane and space, five hours, with exercises, one hour ; Descriptive geometry, with exercises, two hours.

UNIVERSITY OF MUNICH.—By Professor G. BAUER : Algebra ; subject of second course to be announced ; Seminar.—By Professor F. LINDEMANN : Plane analytical geometry ; Introduction to ordinary and partial differential equations ; Mathematical principles of life insurance ; Seminar.—By Professor A. PRINGSHEIM : Differential calculus, with exercises ; Elliptic functions.—By Dr. K. DÖHLEMANN : Descriptive geometry, with exercises ; Kinematics.—By Dr. E. v. WEBER : Introduction to analysis ; Determinants with applications ; Selected chapters of elementary geometry.—By Dr. J. GÖTTLER : Theory and application of the functions of Lamé, Bessel and Laplace.

THE STEINER PRIZE.—The Academy of sciences of Berlin at the Leibniz Meeting, July 4, 1895, announced the following problem for the Steiner Prize: "To completely solve any important, hitherto unsolved problem relating to the theory of curved surfaces, taking into account, so far as possible, the methods and principles evolved by Steiner. It is required that sufficient analytical explanations shall accompany the geometrical investigations to verify the correctness and completeness of the solution. Without wishing to limit the choice of subject, the Academy takes the opportunity to call attention to the special problems to which Steiner has referred in his general remark at the end of his second paper on maximum and minimum in figures in a plane, on a sphere, and in space."

No paper on this subject was received. In accordance

with the terms of the Steiner foundation, the Academy has utilized the prize of 6,000 marks, thus unawarded, for the purpose of recognizing certain important geometrical contributions published during the last few years. One third of it was awarded to each of the following: To DR. KARL FRIEDRICH GEISER, professor at the polytechnic school at Zurich, for his individual researches in geometry and his services in the publication of a part of Steiner's lectures; to DR. DAVID HILBERT, professor at the University of Göttingen, for his important researches on the axioms of geometry and for the advancement which analytic geometry has experienced from his work on the theory of invariants; to DR. FERDINAND LINDEMANN, professor at the University of Munich, who has earned special distinction in geometry by his celebrated discussion of the quadrature of the circle, as well as by editing Clebsch's *Vorlesungen über Geometrie*.

For the year 1905 the Academy again announces the foregoing problem, which has remained unsolved. For its solution a prize of 4,000 marks is offered, with an additional sum of 2,000 marks. Papers offered in competition may be written in German, French, English, Italian or Latin, and must be submitted before December 31, 1904. The result will be announced at the Leibniz meeting of 1905. No manuscript revealing the name of the author will be accepted. Each manuscript is to bear a mark or nom-de-plume, and to be accompanied by a sealed envelope containing the name and address of the author and bearing outside the corresponding mark or assumed name. Manuscripts should be sent to the Bureau of the Academy, Berlin NW. 7, Universitäts-strasse, 8.

FROM an article in *Science*, August 31, 1900, it appears that the degree of doctor of philosophy was conferred by American universities during the past year on 233 candidates, of whom 11 presented dissertations in mathematics. The list of mathematical doctorates, with titles of theses and names of universities conferring the degrees, is as follows: L. P. EISENHART, Johns Hopkins University, "Infinitesimal deformation of surface"; W. GILLESPIE, University of Chicago, "Determination of all hyperelliptic integrals of the first kind of genus 3 reducible to elliptic integrals by transformations of the second and third degrees"; H. E. HAWKES, Yale University, "Examination and extension of Peirce's linear associative algebra"; D. N. LEHMER, University of Chicago, "Asymptotic evaluation of certain totient sums"; J. H. McDONALD, University of Chicago,



“Concerning the system of the binary cubic and quadratic with application to the reduction of hyperelliptic integrals to elliptic integrals by a transformation of order four”; C. R. McINNES, Johns Hopkins University, “Superoscillated sections of surfaces”; H. C. MORENO, Clark University, “On ruled loci in  $n$ -fold space”; F. R. MOULTON, University of Chicago, “A particular class of periodic solutions of the problem of three bodies”; S. E. SLOCUM, Clark University, “On the continuity of groups generated by infinitesimal transformations”; F. B. WILLIAMS, Clark University, “Geometry on ruled quartic surfaces”; A. D. YOCUM, University of Pennsylvania, “An inquiry into the fundamental processes of addition and subtraction.”

THE Hopkins prize of Cambridge University for the period 1894–1897 has been awarded to Mr. J. LARMOR of St. John’s College for his investigation on the “Physics of the æther” and other contributions to mathematical science.

THE works of the late Professor E. BELTRAMI are to be published by the faculty of science of the University of Rome. The edition, which is to be issued by subscription, will consist of three or four large volumes. A complete list of Professor BELTRAMI’s published papers appeared in the *Annali di Matematica* for March, 1900.

THE recent catalogue, No. 127, of M. Hoepli, Milan, Italy, contains nearly three thousand titles of memoirs and treatises in mathematics and physics.

PROFESSOR R. LIPSCHITZ, of Bonn, has been elected a correspondent of the section of geometry and Professor P. DUHEM correspondent of the section of mechanics of the Paris academy of sciences.

PROFESSOR F. KLEIN, of Göttingen, has been elected corresponding member of the Vienna academy of sciences. At the recent celebration of its five-hundredth anniversary, the University of Cracow conferred on Professor Klein an honorary doctorate.

DR. J. H. GORE, professor of mathematics and geodesy at the Columbian University, has been appointed juror-in-chief of the Court of Appeal of the Paris Exposition.

PROFESSOR A. R. FORSYTH has been elected an honorary member of the Edinburgh Royal Society.

DR. ARNOLD EMCH, recently professor of graphic mathematics in the Kansas State Agricultural College, has been appointed to an assistant professorship of mathematics in the University of Colorado.

DR. LOUIS T. MORE, of the University of Nebraska, has been elected professor of physics in the University of Cincinnati.

DR. JACOB WESTLUND, of Yale University, has been appointed instructor in mathematics in Purdue University.

DR. H. FEHR, docent at the University of Geneva, has been promoted to a full professorship in geometry and algebra at the same university.

DR. KARL BOEHM has been made docent in mathematics, at the University of Heidelberg.

MR. W. B. FITE and Mr. H. B. KUHN, recent holders of fellowships, have been appointed assistants in the department of mathematics at Cornell University.

PROFESSOR C. S. VENABLE, emeritus professor of mathematics in the University of Virginia, died August 11, 1900, at his home in Charlottesville, Va., at the age of 73 years. His education was received at Hampden Sidney College and the University of Virginia, and at the Universities of Berlin and Bonn. He served in the Confederate army with distinction throughout the civil war, most of the time as a member of Gen. Lee's staff. Before the war he was successively a member of the faculties of Hampden Sidney College, the University of Georgia, and the College of South Carolina; and since the war he served continuously as professor of mathematics in the University of Virginia until in 1896 failing health induced him to resign.

As the author of a series of well known text books on mathematics and by his work at the University of Virginia, he exerted a profound influence upon mathematical instruction throughout the Southern States. As a pupil of Encke and Argelander, he became deeply interested in astronomy. In 1860 he observed the total eclipse of the sun in Labrador, and more recently it was mainly through his exertions that the establishment and endowment of the Leander McCormick Observatory was secured.

PROFESSOR J. E. KEELER, director of the Lick Observatory, died at San Francisco, August 12, 1900, aged 43 years.

