ematicians in a branch of pure mathematics which becomes the more fascinating the more it is studied,'' and thus accomplish the author's object.*

G. A. MILLER.

CORNELL UNIVERSITY,
March, 1900.

SHORTER NOTICES.

Traité de Nomographie. Par MAURICE D'OCAGNE. Paris, Gauthier-Villars, 1899. ix + 480 pp.

GIVEN a function of several variables, to read off its approximate values for all values of the variables, by simple inspection of a diagram drawn once for all—such is the general problem the investigation of which has led to this attractive volume. The diagram is often provided with travelling parts. The whole apparatus is called an *abacus*.

The problem is primarily a practical one. The technical arts force upon us relations or laws. So soon as a law is of frequent occurrence, its abacus is desirable. Even where great accuracy is required, an abacus is useful for getting a first approximation : for instance in the calculation of annuities (Abaque de M. Prévot), or in the solution of Kepler's equation

$$a - e \sin a = \mu$$

(abaque de M. d'Ocagne).

The scientific question as to what laws are capable of such exhibition is reserved for the final chapter. The rest of the book consists of classified instances. These instances are of quite surprising range and power. Practically a new subject is sprung upon us, claiming to be useful in so many directions that it would strain the faculty of an institute of technology to review the book in full detail. Thus we find an abacus (again by M. Prévot) used in the construction of the chromatic harp, an abacus of the penetration of light from a lighthouse (M. Allard), of the march of troops (M. Goedseels), of the deviation of the compass for an assigned ship (M. Lallemand), of the gauging of yachts (M. Chancel). These few random instances will serve to indicate the possibilities of the abacus in abolishing needless arithmetic. But the applications to the classic problem of

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cuttings and embankments must be especially mentioned, as M. d'Ocagne implies that the subject really grew out of this problem.

In the construction of an abacus, free use is made of the elements of analytic geometry, and in particular of both point and line coördinates. But the curve of the geometer is replaced at the outset by the scale (échelle). That is, the relation $y = f(x)$ is shown not by a curve but by the Y-axis itself, the point of this axis whose distance from the origin is $f(x)$ being marked simply x. Thus the common slide rule is built of two parts, either of which is a scale of logarithms, representing in the manner stated $y = \log_{10} x$.

So a scale of cosines might bear the numbers 0, 10, 20, ..., 180, the number x being distant from the origin $\cos x^0$, or $l \cos x^0$ if $2l$ is the length of the scale.

Such scales play a great part in the construction of abaci. But referring to the book itself for the other means employed, we shall do better to give a specific instance of an abacus.

Consider two points of a sphere, whose latitudes are λ and λ' and the difference of whose longitudes is L. The angle at the center is φ where

$$\begin{aligned} 2 \cos \varphi = {} & (1 + \cos L) \cos (\lambda - \lambda') \\ & - (1 - \cos L) \cos (\lambda + \lambda'). \end{aligned} \tag{1}$$

Write $\quad u = - l \cos (\lambda + \lambda'), \; v = l \cos (\lambda - \lambda')$.

Then $\quad u(1 - \cos L) + v(1 + \cos L) = 2 \, l \cos \varphi. \qquad (2)$

Take a square of side $2l$, whose center is the origin. Mark a scale of cosines along the right side upwards, along the left side downwards, and along the top to the right. From the right scale read the number marked $\lambda - \lambda'$; its ordinate is u. From the left scale read the number $\lambda + \lambda'$; its ordinate is v. These numbers u and v are parallel coördinates of a line (as in Salmon, Conics, p. 386 of the sixth edition). And whatever values we assign to u and v, subject to (1), the line passes through a point; that is, (2) is the equation of a point. By an easy calculation this point is

$$x = l \cos L, \quad y = l \cos \varphi.$$

We lay then a line through the points $\lambda \pm \lambda'$, and observe where it cuts the vertical through the point of the top scale marked L; the ordinate of this point is $l \cos \varphi$, and φ itself is read off from the scale at the right.

The square with its scales, and the moveable line, constitute then the abacus of the equation (1). Naturally, the square is marked with vertical and horizontal lines to assist the reading.

The questions raised in the book should appeal not only to the technical man, but also to the teacher of elementary analytic geometry, at least to those teachers who care to heed that class of students whose cry is "of what use is this?"

FRANK MORLEY.

An Elementary Treatise on the Theory of Equations. By S. M. BARTON, Ph.D. D. C. Heath & Co., Boston, 1899. 8vo, xii + 198 pp.

THE title of Professor Barton's book suggests at once to the English-speaking student of mathematics the well-known treatises of Todhunter and of Burnside and Panton, and calls to mind how consistent English practice has been in assigning the theory of equations to distinctive works on the subject. Among the comparatively few books thus styled may be mentioned Chapman's "An Elementary Course in Theory of Equations," published in this country a few years ago.

The present treatise is much more limited in scope than the English works above mentioned. There is no attempt to deal with the formal side of the higher algebra. The book is intended expressly for undergraduate instruction in our colleges and technical institutions, and the contents and treatment are accordingly quite narrowly prescribed by the requirements of the usual college course.

The work falls into two parts: I., an elementary exposition of determinants; II., the theory of equations proper.

Part I.—The first two chapters give the principal theorems of determinants, and the third consists of applications to linear equations and a consideration of special determinant forms. The subject is introduced by considering the permutations of a group of elements, after which the development follows the usual course. The author here undoubtedly exposes himself to criticism for devoting so much space, some seventy-five pages in all, to a theory of which he is able to make very little use in the remaining chapters.

Part II.—In this part Burnside and Panton have been laid under heavy contribution. An inspection of the table of contents and a review of the pages show, as the author states, that the development has followed very closely in the lines of the first ten chapters of the treatise cited. It