of our senses and of intuition, and the world of number. Objects in the first world give occasion to form certain objects in the world of number which we strive to make as close to the original as possible. How close the copy is we can never know. Doubtless they are sufficiently approximate.

The mathematician of to-day, trained in the school of Weierstrass, is fond of speaking of his science as "die absolut klare Wissenschaft." Any attempts to drag in metaphysical speculations are resented with indignant energy. With almost painful emotions he looks back at the sorry mixture of metaphysics and mathematics which was so common in the last century and at the beginning of this.* The analysis of to-day is indeed a transparent science. Built up on the simple notion of number, its truths are the most solidly established in the whole range of human knowledge. It is, however, not to be overlooked that the price paid for this clearness is appalling, it is total separation from the world of our senses.

YALE UNIVERSITY,
February, 1899.

TWO BOOKS ON TIDES.

The Tides and Kindred Phenomena of the Solar System; the substance of lectures delivered in 1897 at the Lowell Institute, Boston, Mass. By GEORGE HOWARD DARWIN. Houghton, Mifflin & Company, 1898. 8vo, xviii+378 pp.

Leçons sur la Théorie des Marées; professées au Collège de France. Par MAURICE LÉVY. Paris, Gauthier-Villars et Fils, 1898. 4to, xii+298 pp.

IT is not often that the reviewer has the opportunity of noticing a volume containing a popular account of an abstruse and difficult subject set forth by an author who stands in the forefront as an investigator of the matters on which he writes. It is never easy for any one who spends most of his time at the confines of his science to tear him-

* Hamilton, Life, vol. 1, p. 304, writes in a letter dated 1828: "An algebraist who should thus clear away the metaphysical stumbling blocks that beset the entrance to analysis without sacrificing those concise and powerful methods which constitute its essence and its value would perform a useful work and deserve well of science.

self away from what interests him most and to give up a large portion of his time to the writing out of an account of his subject. The task becomes increased when the object in view is a book free from the technical phrases and symbols which enable him to express his ideas clearly and tersely to experts. This task, however, Professor Darwin has undertaken and achieved with a success which would have been noteworthy in any subject, but which is remarkable when the intricate nature of the tidal problem is remembered. Moreover, he has filled a gap in the numerous series of popular handbooks of science. We have called attention before to the need of such a book on hydrodynamics. The deficiency is partly made up by the volume before us and its appearance will make the labor of the writer who undertakes a popular exposition of the general subject a much easier task than it would otherwise have been.

In his Preface the author writes: "I think that there are many who would like to understand the tides and will make an effort to do so provided the exposition be sufficiently simple and clear; it is to such readers that I address this volume." The book has been submitted by the reviewer to one of those whom the author addresses and who was without the mathematical knowledge which makes such accounts comparatively easy reading. It has stood this test well. The first part, which deals almost entirely with tides in seas, rivers, and lakes, has fulfilled its object in being clear and simple and in giving information about the manner in which the tides are produced and the peculiarities of their behavior under different circumstances. The second half of the book, on the effects of tidal friction and theories of cosmogony, was naturally found more difficult; although the matters treated are mainly of scientific interest only, the unskilled reader, if he is willing to take a little trouble, is still able to follow the arguments right to their conclusion and to obtain a general idea of their results.

There is much more, however. In addition to a general survey of the whole subject, the results of the latest investigations (*e. g.*, on rotating figures of equilibrium and the evolution of stellar systems) are given with some detail. Fully two thirds of the volume contains matters which have never appeared outside the scientific journals in which they were originally printed. We have practically a complete bibliography in the references given at the end of each chapter. Finally the author frequently points out the deficiencies in our knowledge and especially the problems

which seem to be capable of being solved but which have not hitherto yielded to analysis. In fact, there are many suggestions for those who desire to take up the subject from the mathematical side.

The first three chapters contain general accounts of the observed phenomena and of the instruments used for recording them. Considerable space is devoted to an important but little known class—the vibrations of water in lakes, usually called seiches. Under this heading general wave motion at the surface of a fluid is explained. The distinction between rise and flow and between ebb and fall in a river or estuary are carefully set forth. These terms are often considered synonymous; it is not easy for most people to grasp the idea that the water may be rising while the current is flowing seaward. It would have been advisable to give more explanations as to why, in a river of uniform depth and breadth at a great distance from the mouth, the current ebbs at a given place when the water there is below mean level and rises when it is above mean level, with slack water at mean level. In wave motion the particles of water describe closed curves which, when the waves are on deep water, are approximately ovals with their longer axes vertical. In shallow water these ovals will tend to have their longer axes horizontal, and, with the very long tide wave in a river, the ratio of the axes becomes that of some miles to a few feet. It is then easy to see how we may have a flow when the water is above mean level and an ebb tide when it is below. An interesting account of the bore on the Tsien-Tang-Kiang forms a large part of Chapter III. Chapter IV is historical.

In Chapter V it is explained in what way the sun and moon produce the tides. The author shows how the tide generating forces vary inversely as the cube of the distance and directly as the product of the masses and also that it is their horizontal and not their vertical components which produce the tides. From the mathematical point of view this is the most successful part of the book. We are thence led on to consider the deflecting effect of such forces on the pendulum and the elastic distortion of the earth's surface by the varying tidal load. The attempt of George and Horace Darwin to measure the lunar attraction on a pendulum is given with some detail and will be read with special interest by those who have attempted to make delicate measurements near a modern city.

Chapters VIII and IX respectively contain accounts of Laplace's equilibrium theory and Airy's canal theory of

the tides. In the former, the principal lunar and solar tides are described and it is shown how the combined action of the sun and moon produces spring and neap tides. In the latter, the author explains the difference between free and forced waves and the peculiarities of the forced waves, in order to show how we may have a low tide under the moon, directly contrary to the result of the equilibrium theory. Then follow chapters on tides in landlocked seas, the methods of separating out the composite tide into its several component parts, the mathematical and mechanical reduction of tidal observations for finding the constants of a port; the last appears for the first time in a form which is easily comprehended without referring to the author's original memoirs on the subject. Chapter XIV, on the degree of accuracy of tidal prediction, closes the portion of the book devoted to tides, in the popular sense of the word.

Professor Darwin is perhaps better known to the outside public for his work on the past and future history of the moon. It was he who first showed the tremendous influence which the tides of a fluid or semi-viscous spheroid can have in modifying the relations between two celestial bodies revolving about one another. In the later chapters of his book we are given an account of the results of his investigations and of those who have followed in his steps. He begins by a discussion of the arguments for and against the solidity of the interior of the earth, opening with a description of the nutation discovered by Chandler. The latter would have been rendered clearer by a fuller explanation of precession and the ordinary nutation: it is not quite evident in the example of the top whether its precessional or nutational motion is under consideration. The degree of rigidity of the earth is an important factor in tidal problems investigated from the theoretical point of view. The question has not yet been answered by analysis; the author quotes Lord Kelvin's opinion that the tides, if the earth were rigid, would not be twice as great as they actually are.

The next two chapters are the most fascinating in the book; they deal with tidal friction and lead up to the consideration of the past and future history of the earth and moon. Professor Darwin first explains in detail how tidal friction affects the time of rotation of the earth about its axis and that of the moon about the earth; his geometrical demonstration is clear and easily made out. We are led by simple steps to the time in the past when the lengths of the day and month were equal and very short, and after-

wards to the future date when they will be again equal but comparatively long. The causes making for modification of these results are also carefully stated so that the reader does not run away with the idea that he is obtaining actual facts; on the contrary, a theory is put before him which cannot indeed be tested but for which certain arguments give a fair degree of justification. The theory is sufficient to explain the main relations of the earth-moon system.

Having carried the theory to a stage in the past history when the two bodies had very probably not yet become solid, the author prepares the way for a step farther back by considering the equilibrium of a mass of rotating fluid. The question of the stability of Maclaurin's and Jacobi's ellipsoids is explained and the essence of his own work and that of Poincaré on the formation of two masses of rotating fluid from one is given. We are thus naturally led to the nebular hypothesis and the evolution of celestial systems generally, these being considered in Chapter XIX. The evidence for this hypothesis furnished by double stars, the increasing number of satellites possessed by the planets of our solar system as we go outward from the sun, and by the forms of the nebulæ visible in the sky, are examined. A whole chapter—the last—is devoted to an examination of the bearing of Saturn's rings on the theories. The researches of Roche and Maxwell are graphically described and it is shown how the rings furnish a strong link in the chain of evidence which goes to justify the position taken up.

All these matters the author has been able to explain with but few tedious circumlocutions and without the use of a single algebraical symbol, except in one short footnote where, on a minor detail, a little elementary algebra is used. And he has wisely done away with the lecture form necessary before an audience but exceedingly tiresome to the reader. Finally, those who have had the privilege of hearing his lectures will be glad that the suggestiveness which is one of their chief attractions, is retained in this volume and thus can give inspiration to a wider circle.

The type and printing are excellent and the illustrations good, with the exception of the pictures of the bore on the Tsien-Tang-Kiang, which seem somewhat rough. There are very few errors. On p. 272, line 8, "neither" is written for "either." The following have been communicated to the writer and may be also recorded for the sake of future readers. On p. 29, line 11, "at right angles" should be "along." On pp. 139, 140, the words "right" and "left"

should be interchanged to make the text agree with the figure. On p. 189, line 25, "lines of equal establishment" should be "lines of Greenwich mean time of high water."

It is not altogether easy to decide from what point of view a volume like M. Lévy's should be treated. Its title would lead us to expect the usual publication of a lecture course with the additions and suppressions necessary to make the exposition more complete and continuous. When one reads the preface and table of contents, however, it appears that the author has desired to do away with any traces of the lecture room in order to put together a connected treatise : this impression is confirmed on reading the text and noticing the several tables which he gives. It aims at a rather complete presentation of the theory of the tides and, as it happens, will form an excellent mathematical commentary on Professor Darwin's book. The first part, which forms the subject of this notice, contains the equilibrium theory of Laplace, the methods of harmonic analysis and tidal prediction initiated by Laplace, Kelvin, and Darwin, the canal theory of Airy, and the general theory of tides in estuaries and rivers ; at the end a chapter is devoted to the solitary wave. A second part is promised dealing with the interesting subject of the socalled "solid" tides—those oscillations of the earth's crust and of its whole mass which as one result have led to arguments concerning the solidity of our globe.

The various parts of the volume are of rather unequal merit. When the author is explaining the more theoretical parts of his subject and showing their applications to the phenomena actually observed, he does so with all the lucidity which past experience has led us to expect from French scientists. These parts fortunately form the larger portion of the book ; the demonstrations are easily followed and the results as easily grasped. But in the chapters dealing with harmonic analysis and tidal prediction the matter seems to suffer from having been insufficiently digested. Whole sections and chapters have been taken straight from Darwin's memoirs with a few changes of arrangement and the insertion here and there of algebraical steps. Such a procedure is seldom satisfactory. A memoir is primarily addressed to those who are acquainted with the previous work which has been done and its matter is not usually in a form suitable to make it part of a treatise. Moreover, too much seems to have been inserted for a clear comprehension of the methods, while any one wishing to make practical use

of them would probably be obliged to have recourse to the original sources. However this may be, M. Lévy's work will rightly receive a hearty welcome. So far as we know, it is the first publication of a treatise specially devoted to the mathematical theory of the tides, written for the use of students. The author has shown excellent judgment in his selection of the more important parts of the subject. He never loses sight of the practical applications; almost every result is discussed in an interesting way and its relation to some particular or general tide shown.

Chapter I contains the equilibrium theory with Lord Kelvin's extension for taking into account the forms of the continents. Laplace's method for the prediction of tides, as given in the Mécanique Céleste, forms the subject of Chapter II, and the author here takes the opportunity of explaining the principal solar and lunar tides, the "establishment" of a port, the use of tide tables, etc. The next four chapters, on the development of the tide generating potential, the harmonic analysis of the observations, the determination of the constants of a port, and the latest method of forming tide tables, are mainly taken from the B. A. report of Adams and Darwin of 1883, and the Royal Society Memoirs of the latter for 1890. The first of these is well put and reads simply, notwithstanding the long formulæ which occur in the development of the potential function. The other three chapters forming the rest of the first section have been referred to above.

The method introduced by Airy of treating the tidal problem as a dynamical rather than a statical one is contained in Chapter VII. In the equilibrium theory the water, at every instant, is supposed to take up the position of equilibrium which it would have if the tide raising body, the earth and the water had been relatively fixed for that instant: this practically amounts to neglecting the relative inertia of the fluid. There are thus no differential equations to be solved. In the correct dynamical theory, the partial differential equations arising present difficulties which can only be surmounted in special cases. Those considered by Airy are the tides in canals which follow great or small circles on the earth's surface. In his account of Airy's method, M. Lévy has introduced the friction of the water on the bed of the canal at all stages and the effects are worked out with some detail. Most of the special applications are to be found in Chapter VIII where he treats cases of canals limited at the ends, those communicating with lakes or seas, etc. One of some interest is the com-

parison of the results of theory and observation for a part of the Suez canal which communicates at one end with a tidal sea and at the other with small lakes : the differences are not greater than might have been expected.

Chapter IX will be read with interest by those who have taken the trouble to observe the eccentricities of the currents in a tidal river or estuary. Here it is mainly a question, not of the tides produced directly in the estuary by the sun and moon, but of the oscillations due to the rise and fall of the ocean at the mouth. The relation of the height of the water at any point of an estuary to the current at the same place is well and fully worked out. St. Venant's particular solution for a river of constant width and indefinitely prolonged towards the source, which gives the current as a function of the height only, receives considerable attention. The author shows how a defect in the practical application of it can be remedied by introducing friction ; the calculations, however, become rather complicated. The chapter closes with the problem of a river whose width varies according to an exponential law. In Chapter X, under the heading of the solitary wave, the author treats standing waves and the propagation of a wave or hollow of given form ; he follows the methods of Boussinesq entirely, omitting to mention some of the later investigations. The question of the stability of form of the wave is included.

The book, as a whole, is to be highly recommended and it will form a useful addition to the literature of hydrodynamics. The name of Gauthier-Villars is a sufficient indication of the excellence of the printing.

ERNEST W. BROWN.

NOTES.

BEGINNING with Volume XIII the *Annals of Mathematics* will be published under the auspices of Harvard University. A circular, about to be issued by the Department of Mathematics at Harvard, furnishes the following statement in regard to the conduct of the *Annale* under the new auspices:

"There has existed in this country for more than twenty years a journal (the *American Journal of Mathematics*) devoted almost exclusively to original research, and the *Transactions of the American Mathematical Society* will soon give still