

Weingarten in particular, Darboux concludes the volume by determining all triple systems for which the nine quantities H_i, β_{ik} depend on a single variable a ; the result is a system composed exclusively of helicoids having constant total curvature.

EDGAR ODELL LOVETT.

PRINCETON, NEW JERSEY.

THE NEW MATHEMATICAL ENCYCLOPAEDIA.

It is an established belief that mathematics like the classics stopped growing long ago. The chemist, biologist, physicist, and other scientists look with complacency at the gigantic strides their branches of learning have taken in the last generation and rather pity their colleagues, the mathematicians. According to them, the golden age of our science dates back two thousand years ago when Euclid, Archimedes, Apollonius and Diophantes flourished. It is true that some time afterwards algebra and trigonometry, analytical geometry and the calculus were invented; they have, however, been long since perfected and mathematicians spend their time teaching this ancient body of facts, improving here and there a small detail and solving ingenious problems which they devise to test each other's skill. How different the real state of affairs is! No science presents a more intensely active and vigorous condition than ours. Indeed the growth of mathematics has been so rapid in the last century, the discoveries and inventions so numerous and their importance so far reaching that it is permitted only to a very few extraordinary minds still to overlook the whole field of mathematics as it stands to-day. One has only to recall a few of the great theories which have sprung up in these later days and the numberless special theories and ramifications they have given rise to. First of all that great leviathan, the function theory, embracing as special topics of limitless extent the theory of elliptic, hyperelliptic, abelian, modular, and automorphic functions not to mention others. Of still younger date is the theory of groups finite and infinite, with their far-reaching applications. We have then the modern theory of invariants, the new theory of algebraic numbers, the non-Euclidian geometry, the theory of algebraic transformations and correspondences, Cantor's theory of multiplicities, the partial

differential equations of the potential theory, the modern theory of linear differential equations inaugurated by Riemann and Fuchs, the arithmetical theories of Kronecker, etc., etc.

But there is no need to enumerate further. The field of mathematical activity is so boundless, the details so rich and varied that the ordinary mind falls into a state of helpless despair when occasionally contemplating it as a whole. The days when, with industry and patient perseverance, one could embrace all that was known in mathematics or even all that is fundamental and important have long since passed by. But a more hopeless state of affairs has begun to prevail; we not only cannot embrace the whole but the most of us cannot even keep up with the latest advances in a few special theories. We find ourselves compelled more and more to limit the range of our vision, to confine ourselves even to a part of a single theory. Nor is this wonderful when we reflect how rapidly mathematical literature is increasing. In the course of a single year 2,000 books and memoirs on mathematical subjects appear, while the number which have appeared during the last generation is estimated at about 50,000.

Specialism is thus the motto of the day, it is not a matter of choice but of necessity. But against this state of bondage two forces are constantly at work. In the first place it is an inherent, innate principle of the human mind to have a lively interest in what immediately surrounds it, and this is true in the world of mathematics as everywhere else. The normal mathematician cannot help longing to know something of the regions which border on his own special field of study. He is as eager to travel through some of these unknown countries, admiring their peculiar beauties and acquiring fresh insight into the deeper relations of things, as any tourist for a jaunt through India or Zululand.

This is the ideal force; the other force is of very practical and urgent nature. Every mathematician knows that the objects of the mathematical world hang together more or less intimately. No theory stands isolated from all others. The systematic attempt of purists to banish all imagined foreign elements from their theories must lead to utter sterility and self destruction. No mathematician can afford to be altogether a specialist. Indeed are we not all of us from time to time forced to leave our chosen field of research and pick up as rapidly as possible the essentials of some widely different theory in order to overcome a difficulty in our path, or to read an important paper engendered

by a different circle of ideas than our own? Often we find it impossible to do this. The facts we are in need of are of so special a nature, they lie so embedded in a mass of complicated concepts and theories that the labor of unraveling them from their surroundings is too great to be undertaken, at least for the present. That this is a grave misfortune is manifest; that it must occur to each individual mathematician more and more frequently as our science becomes not only broader but also more specialized is equally patent. In this dilemma mathematicians have sought assistance in several directions. In the first place attempts have been made to make existent knowledge more accessible by having exhaustive and up to date bibliographies prepared. I mention here the "Répertoire bibliographique des sciences mathématiques" appearing under the auspices of the Société Mathématique de France. In the second place the Deutsche Mathematiker-Vereinigung, with characteristic energy, has instituted a series of admirably prepared reports on the various departments of mathematics. Already reports on invariants, algebraic functions and their integrals, algebraic numbers, and synthetic geometry have appeared and many others are in preparation. These reports are historical.* Going back to the first germs, the development of the theory in all its principal branches is traced down to the present day. They presuppose upon the part of the reader more or less familiarity with the modern state of the particular theory in question; their object is to give him a more comprehensive and deeper knowledge of it by showing him how the various branches of the theory arose, how each was related to and influenced by its surroundings, and how it in turn reacted on them. Hardly a science exists where it is so important to have a historical knowledge of the subject as in mathematics.

Both these undertakings fill a long felt want. But there exists still another need more pressing, more constantly felt than either of these. It is the need of a mathematical encyclopedia. Attempts to meet this crying demand have already been made. We have in German Hoffmann and Natani's *Mathematisches Wörterbuch* in seven volumes, (Berlin, 1858-67) Schlömilch's *Handbuch der Mathematik* in two volumes, (Breslau, 1879-81). In English we have Carr's *Synopsis of Mathematics* in one volume, (London, 1886). None of these are of much service to the professional mathematician of today. Finally we have a work

* Hilbert's report is an exception.

of much more advanced character and consequently of greater usefulness in J. G. Hagen's *Synopsis der höheren Mathematik*, of which two volumes in large quarto have appeared (Berlin, 1891-94). A third and final volume will appear shortly. To give an account of mathematical theories as they stand today unfinished and scattered in a thousand memoirs is of course utterly beyond the powers of a single man. Father Hagen has accordingly limited his undertaking to a digest of the principal treatises, and to bibliographical references confined for the most part to those given in these works.

But to fully satisfy the demands of the mathematician of today an undertaking on a far larger scale is necessary. A synopsis based on treatises must necessarily be years behind the present state of the science; on the other hand to prepare a work which shall give the latest developments of our science is not the labor of one man but of scores. To have each subject adequately treated requires that each contributor should have made his subject one of special study and research; he must be familiar with all its phases, its multitudinous ramifications, its history and literature. Such men are difficult to acquire. They are already overburdened partly by their own original investigations, partly by their university duties. To write one of these articles would be a great sacrifice. In the case of our great popular literary encyclopaedias like the *Encyclopaedia Britannica* or *Brockhaus's Conversationslexikon* the sales are so large that the contributors receive a comfortable compensation for their labor; but this cannot be calculated upon for a mathematical encyclopaedia. Pecuniary compensation must be nominal. Besides the contributors there must be an efficient board of editors. Their labors are still more arduous, the sacrifice of their time proportionately greater. Still another important factor must be considered. Since the sales of such a highly technical work must necessarily be relatively limited, what publisher would be willing to assume a risk whose financial prospects were so problematical? Under these unpropitious circumstances it might well seem that a mathematical encyclopaedia as here sketched would never be more than a dream.

That to-day, however, the *Encyclopaedia* is indeed a fact, that already a first Heft has appeared and others are to follow in rapid succession is largely due to the genius, energy and courage of a single man, Felix Klein. Who besides him could set springs in action which would bring together in enthusiastic coöperation two score or more of specialists

from all countries to contribute articles, find efficient editors to unify and make ready for the press the mass of material sent in, and finally by winning the support of the academies of Göttingen, Munich, and Vienna guarantee to a certain extent the financial success of the undertaking to the publishers?

Having shown how great the need of such an encyclopaedia is, and indicated some of the difficulties to be overcome in its realization we wish now to give a few details which will enable the readers of this BULLETIN to form a more accurate idea of this great work. According to what scheme is the work planned, who are the men entrusted with its execution and what are the names of some of the principal contributors?

In the preface to the first number we read: "The object of the Encyclopaedia is to give in a concise form, adapted to rapid orientation, but with all possible completeness, an account of mathematical sciences as they exist today, together with carefully prepared bibliographical notices which sketch their developments since the beginning of the present century. Not only pure mathematics, but also its applications to physics, astronomy, geodesy, as well as the various technical branches will be treated, the idea here being on the one hand to show the mathematician what problems the applications give rise to, on the other to inform the astronomer, the physicist, the engineer, what answer mathematics gives to the same." Demonstrations will not be given, at most only sketched; only a limited knowledge of a given subject will be presupposed. The work is to appear in six volumes. Vol. I will embrace arithmetic, algebra, theory of numbers, probabilities and the adjustment of observations, finite differences, numerical calculations. Vol. II is devoted to the theory of functions of real and complex variables, including such subjects as the differential and integral calculus, differential equations, continuous groups, calculus of variations, general function theory, spherical harmonics, Abelian and automorphic functions, etc. Vol. III deals with geometry; topics are pure geometry, the foundations of the applications of algebra and analysis to geometry, algebraic geometry, differential geometry. Vols. IV and V are to embrace the applications of mathematics, while Vol. VI will be devoted to historical, philosophical and pedagogical questions. It will also contain a comprehensive index of the whole work. So much for the plan of the work. Regarding its execution we must be brief. The editors of the Encyclopaedia are H. Burkhardt and W.

F. Meyer; assisting them is a commission appointed by the academies already mentioned consisting of F. Klein, G. v. Escherich, H. Weber, L. Boltzmann, and W. Dyck. To give an idea of the kind of men who are contributing articles we mention *quite at random* the names: Bachmann, Burkhardt, Castelnuovo, Dyck, Enriques, Fricke, Hilbert, Hölder, F. Klein, Kneser, Krazer, v. Mangoldt, Maurer, W. F. Meyer, Netto, Painlevé, Pincherle, Pringsheim, Runge, G. Scheffers, Schönflies, Segre, Sommerfeld, Staedel, Study, H. Weber, E. v. Weber, Wirtinger and Zeuthen. To this list of names Americans will be gratified to see added those of M. Bôcher, J. Harkness and W. F. Osgood.

We close wishing the Encyclopaedia all success. May it become a familiar handbook to every American mathematician. In our opinion no mathematical work has ever appeared which has such unquestionable claims to be found in the library of every mathematician. No one can make constant use of its wealth of information without deriving invaluable assistance. We have no doubt it will enjoy the most widespread popularity not only with mathematicians but with physicists, astronomers, and engineers in every land. May Americans be quick to avail themselves of this veritable vade-mecum.

JAMES PIERPONT.

YALE UNIVERSITY,
December, 1898.

ERRATA.

VARIOUS errata occurring in previous numbers of the present volume of the BULLETIN are herewith noted:

On page 2, line 10, the name of Professor W. E. Story should be inserted in the list of members present at the Summer Meeting.

On page 48, line 19, *for* Sadlerian *read* Sedleian.

On page 100, line 33, *for* not known *read* now known.

On page 156, lines 31-35, a regrettable error occurs, arising from mistaken identity. Mr. Horace T. Eddy was appointed instructor in electrical engineering and physics at Union College at the beginning of the present academic year. Professor Henry T. Eddy continues to fill the chair of engineering and mechanics in the University of Minnesota.