

## NOTE ON NAPIER'S RULES OF CIRCULAR PARTS.

BY PROFESSOR EDGAR ODELL LOVETT.

THE object of this note is to call attention to Napier's conception of Napier's rules of circular parts in the theory of the right spherical triangle. Text-books and treatises are considerably at variance as to the importance and status of these rules. They are continually referred to as mere mnemonics requiring independent proof in each case; and not infrequently their rôle in this capacity is discouraged, analogies with corresponding formulæ in the plane being offered as preferable substitutes. What follows shows that these much abused individuals are entitled to more generous consideration and should be invested with the dignity of a theorem. The circumstance of their various misinterpretations emphasizes again that the student of mathematics should trust no middle man, but go with his own head to original sources, to the masters themselves. Second-hand ideas are as full of bacteria as are second-hand books and clothes.

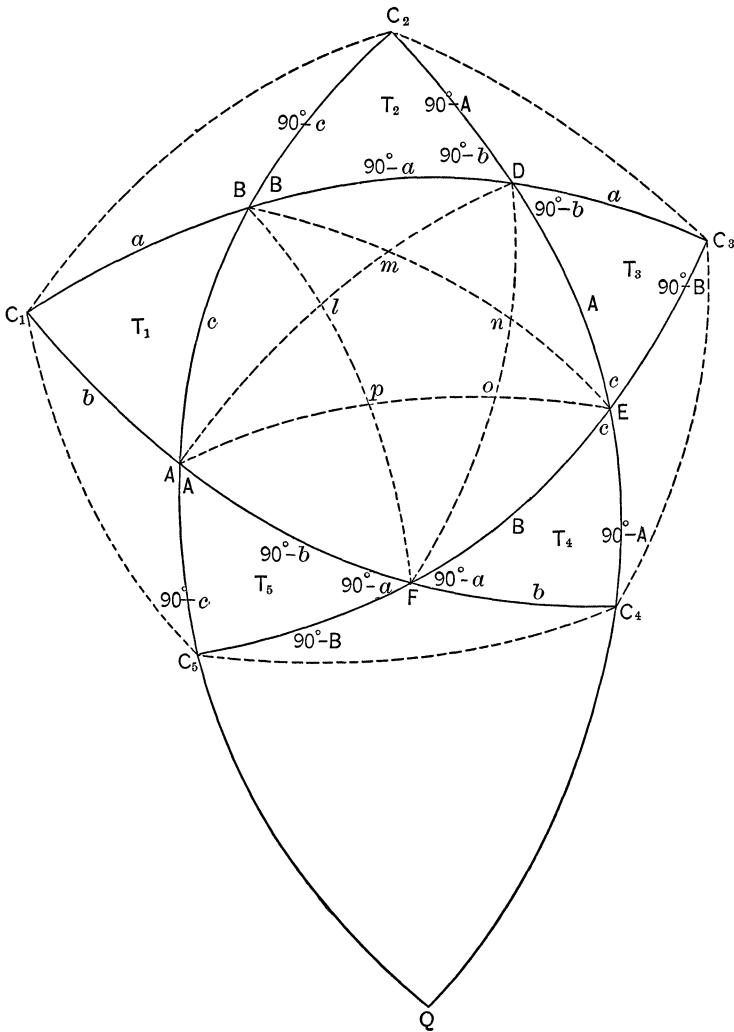
Let  $ABC_1$  be a right spherical triangle whose right angle is at  $C_1$ ; draw the great circles which coincide with the sides of the triangle and those whose poles are the vertices  $A$  and  $B$ . These five great circles form a spherical pentagon surmounted by five right spherical triangles. The uniformity of Napier's rules lies in the fact that all five of these associated triangles, though possessing different natural parts, have one and the same set of circular parts.

A little attention to the geometry of this beautiful figure yields the following geometrical interpretations of the rules of Napier, the first two of which were given by Napier in the original presentation of the rules,\* and the first of which was rediscovered by Ellis.†

Napier's rules express  $1^\circ$  properties common to the members of the system of five right-angles  $T_1, T_2, T_3, T_4, T_5$ ;  $2^\circ$  characteristics which the individuals of the family of five quadrantal triangles  $ABD, \dots$ , have in common;  $3^\circ$  relations among the parts of the self-polar pentagon  $ABDEF$ ;  $4^\circ$  relations in the elements of the pentagon  $C_1C_2C_3C_4C_5$ ;  $5^\circ$  relations among the sides and angles of the pentagon

\* See Napier—*Logarithmorum canonicis descriptio*, Lugduni, apud Barth. Vincentium, MDCXX, Lib. II., Cap. IV., pp. 31 et seq.

† See *The mathematical and other writings of Robert Leslie Ellis*, edited by William Walton, Cambridge, 1863, pp. 328 et seq.



*lmnop*; 6° properties peculiar to the quadrilaterals of the system of five trirectangular quadrilaterals  $FC_5QC_4, \dots$ ; 7° relations common to the members of the family of five complete quadrilaterals  $ABC_2DEm, \dots$ , the latter family including as particular cases the system  $ABDE, \dots$ ;  $ADEB, \dots$ ;  $\dots$ .

It will be further remarked that any one figure of the en-

semble determines all the other members completely, and that the degenerate forms of the aggregate when the original triangle is birectangular or trirectangular are interesting.

Maudit's rules and those of our countryman, Nathaniel Bowditch, are susceptible of similar extensions and interpretations.

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#### NOTES.

THE Colloquium in connection with the Fifth Summer Meeting of the AMERICAN MATHEMATICAL SOCIETY will be held at Harvard University, Cambridge, Mass., during the week beginning Monday, August 22d. The course of lectures offered by Professor Osgood is entitled: "On some methods and problems of the general theory of functions." The title of Professor Webster's course is: "The partial differential equations connected with wave propagation." Details in regard to the Colloquium are contained in the circular issued by the committee in charge.

THE preliminary programme for the coming meeting of Section A of the American Association for the Advancement of Science comprises thirty papers, including several reports on progress in different branches of mathematics. The meeting of the Association takes place August 22-27. Professor E. E. Barnard is the Vice-President, and Professor J. McMahon the Acting Secretary of Section A.

THE Göttingen Royal Society has taken active measures to resume and complete the publication of Gauss' collected works, the first six volumes of which appeared more than twenty years ago. The *Nachrichten* (No. 1, 1898) contains an account, by Professor F. Klein, of the plan in contemplation. The death of Professor Schering, to whom the publication was originally entrusted, has necessitated a distribution of the remaining work among a considerable number of editors. The astronomical material has been assigned to Professor M. Brendel, who also undertakes the general editorial supervision of the whole work. The other editors and their subjects are: Professor R. Fricke, analysis and the theory of numbers; Professor P. Stäckel, geometry; Professors Börsch and Krüger, geodesy; Professor Wiechert, mathematical physics. The work is to be completed in three volumes and a supplementary volume. Vol. 7 will be de-